

**NEP 2020**

**Syllabus of**

**Two-Years Integrated Programme**

**in Mathematics**



**Bodoland University**

**Kokrajhar-783370, Assam, India**

**STRUCTURE OF THE SYLLABUS**  
**TWO-YEARS POST GRADUATE COURSE IN MATHEMATICS**  
**BODOLAND UNIVERSITY**

	Class	Paper	Topics	Credit	Marks
<b>1<sup>ST</sup> YEAR</b>	<b>SEMESTER-I</b>	MATADL 14014	Analysis-I	4	Internal: 30 Theory: 70
		MATADL14024	Functional Analysis	4	Internal: 30 Theory: 70
		MATADL 14034	Partial Differential Equation	4	Internal: 30 Theory: 70
		MATADL14044	Tensor Analysis	4	Internal: 30 Theory: 70
		MATADL14054	Advanced Algebra	4	Internal: 30 Theory: 70
	<b>SEMESTER -II</b>	MATADL 14064	Mathematical Methods	4	Internal: 30 Theory: 70
		MATSPL 15074 (A)	Analysis-II	4	Internal: 30 Theory: 70
		MATSPL 15074 (B)	Mathematical Modeling		
		MATSPL 15084 (A)	Graph Theory I	4	Internal: 30 Theory: 70
		MATSPL 15084 (B)	Continuum Mechanics		
		MATSPL 15094 (A)	General Topology	4	Internal: 30 Theory: 70
		MATSPL 15094 (B)	Differential Geometry		
		MATSPL15104	Computer Lab I	4	Internal: 30 Practical: 70

	Class	Paper	Topics		Credit	Marks
<b>2<sup>ND</sup> YEAR</b>	<b>SEMESTER -III</b>	MATSPL 25014	MATSPL 25014 (A)	Fuzzy Set Theory	4	Internal: 30 Theory: 70
			MATSPL 25014 (B)	Fluid Dynamics I		
			MATSPL 25014 (C)	Advanced Topology I		
		MATSPL 25024	MATSPL 25024 (A)	Graph Theory II	4	Internal: 30 Theory:70
			MATSPL 25024 (B)	Special Theory of Relativity		
			MATSPL 25024 (C)	Category Theory I		
		MATSPL 25034	MATSPL 25034 (A)	Number Theory I	4	Internal: 30 Theory: 70
			MATSPL 25034 (B)	Dynamical Systems I		
		MATSPL 25044	MATSPL 25044 (A)	Advanced Functional Analysis	4	Internal: 30 Theory: 70
	MATSPL 25044 (B)		Numerical Analysis I			
	MATSPL 25054	Computer Lab II		4	Internal: 30 Practical: 70	
	<b>SEMESTER -IV</b>	MATSPL 25064	MATSPL 25064(A)	Fuzzy Logic and Control System	4	Internal: 30 Theory: 70
			MATSPL 25064(B)	Numerical Analysis II		
			MATSPL 25064(C)	Advanced Topology II		
		MATSPL 25074	MATSPL25074 (A)	Graph Theory III	4	Internal: 30 Theory: 70
			MATSPL25074 (B)	Relativity and Cosmology		
			MATSPL25074 (C)	Category Theory II		
		MATSPL 25084	MATSPL 25084 (A)	Number Theory II	4	Internal: 30 Theory: 70
MATSPL 25084 (B)			Dynamical Systems II			
MATSPL 25094		MATSPL 25094 (A)	Analysis III	4	Internal: 30 Theory: 70	
		MATSPL 25094 (B)	Biomathematics			
MATSPL 25104		MATSPL 25104 (A)	Network Theory	4	Internal: 30 Theory: 70	
		MATSPL 25104 (B)	Fluid Dynamics II			

## Detailed Syllabus

### SEMESTER - I

**Paper Code: MATADL 14014**

**Paper Title: Analysis-I**

**Theory Marks: 70**

**Internal Marks: 30**

**Course Outcome:** Comparison of Infinite sets in terms of cardinality, various properties of sequences and series of real-valued functions, properties of continuous real-valued functions on an arbitrary set through the notions compactness and connectedness respectively.

**Unit-I: Marks: 15**

Elements of set theory, finite and infinite sets, cardinal numbers, countable and uncountable sets, Axiom of choice, Real number system.

**Unit-II: Marks: 20**

Sequences and series of real-valued functions, point wise and uniform convergence, Cauchy criterion for uniform convergence, relation of uniform convergence with continuity, differentiation and integration, Weierstrass approximation theorem.

**Unit-III: Marks: 20**

Compactness, Sequential compactness in Metric spaces, Heine Borel Theorem, Bolzano-Weierstrass Property, Totally bounded metric spaces, finite intersection property. Continuous functions on compact metric spaces, Baire's Category Theorem for complete metric space. Arzela Ascoli Theorem.

**Unit-IV: Marks: 15**

Connectedness in Metric spaces, intermediate value theorem, Continuous functions on connected metric spaces, Generalised Intermediate-Value Theorem, Components of a Metric Space.

#### **Text Books:**

- [1] N. L. Carothers. Real Analysis, Cambridge University Press, UK, 2000.
- [2] S. C. Malik and Savita Arora, Mathematical Analysis, New Age International Publishers, New Delhi, 2011

#### **Reference Books:**

- [1] Apostol, T. M. Mathematical Analysis, Narosa Publishing House, 1985.
- [2] Simmons, G. F. Introduction to Topology and Modern Analysis, Tata McGraw Hill Book Co. Ltd., 1963.

## **SEMESTER - I**

**Paper Code: MATADL 14024**  
**Paper Title: Functional Analysis**  
**Theory Marks: 70**  
**Internal Marks: 30**

**Course Outcome:** This study can help the students to learn about basic concepts of functional analysis and can understand about Normed linear space, Banach spaces,  $l^p$  spaces, inner product space.

**Unit-I: Marks: 25**

Normed linear space, Banach spaces, Euclidean and Unitary spaces,  $l^p$ .spaces, Function space  $C[a, b]$ , Subspaces of a Banach space, Convergence and completeness in normed spaces, Completion theorem, Properties of finite dimensional normed linear spaces and compactness, Equivalent norms, Riesz lemma, Bounded and continuous linear operators, Bounded and continuous linear functionals, Normed spaces of operators, Dual space, the dual spaces of  $l^1$  and  $l^p$ .

**Unit-II: Marks: 30**

Inner product spaces: inner product space, Hilbert space, Gram-Schmidt orthogonalization process, Schwarz inequality, triangle inequality, continuity of inner product, polarization identity and parallelogram law, orthogonality, Direct sum and orthogonal complement, orthogonal projection, orthonormal sets and sequences, Bessel's inequality, convergence of orthonormal sequences, total orthonormal sets and sequences, Seperable Hilbert spaces, Riesz's Representation theorem, Hilbert adjoint operators and their properties, self-adjoint, normal, unitary operators and their properties.

**Unit-III Marks: 15**

Hahn-Banach theorem and its consequences, Adjoint operators on Banach spaces, Relation between adjoint operators in Banach and Hilbert spaces, Reflexive spaces, Uniform Boundedness Theorem, open mapping theorem, closed graph theorem.

**Text Books:**

**Text Books:**

- [1] Kreyszig E., Introductory Functional Analysis with Applications (John Wiley and Sons, New York, 1978).

**Reference Books:**

- [1] Limaye, B. V. Functional Analysis (Wiley Eastern Ltd., New Delhi, 1989).
- [2] Rudin, W. Functional Analysis (McGraw Hill, 2000).
- [3] Halmos, P. R., Linear Algebra Problem Book, The Mathematical Association of America (MAA), USA, 1995.
- [4] Halmos, P. R., Finite dimensional vector spaces, Springer Verlag, New York, 1987.
- [5] Simmons, G. F. Introduction to Topology and Modern Analysis (Tata McGraw Hill Book Co. Ltd.,1963)

## **SEMESTER - I**

**Paper Code: MATADL 14034**  
**Paper Title: Partial Differential Equation**  
**Theory Marks: 70**  
**Internal Marks: 30**

### **Course Learning Objectives:**

1. Comprehend the fundamental concepts and origins of partial differential equations, focusing on both first-order and second-order equations.
2. Develop skills to solve various types of partial differential equations, including linear and nonlinear equations using different methods and techniques.
3. Apply analytical methods such as the method of characteristics, separation of variables, and integral transforms to find solutions to partial differential equations.
4. Formulate and solve boundary value problems and initial value problems associated with Laplace's equation, wave equation, and diffusion equation.
5. Relate the theoretical concepts of partial differential equations to practical applications in physics, engineering, and other scientific fields.

### **Course Learning Outcomes:**

1. Demonstrate the ability to solve first-order partial differential equations using techniques such as Cauchy's method of characteristics and Jacobi's method.
2. Effectively solve second-order partial differential equations with constant and variable coefficients, and apply separation of variables and integral transforms.
3. Analyze and solve Laplace's equation for different boundary conditions, and utilize the theory of Green's function to address boundary value problems.
4. Formulate and solve wave equations in one, two, and three dimensions, and understand the application of the calculus of variations in solving related problems.
5. Apply analytical techniques to solve diffusion equations, including the use of separation of variables, integral transforms, and Green's functions to find solutions.

### **Unit-I: Partial Differential Equation of the first order**

**Marks: 15**

Partial Differential Equations – Origins of First Order Differential Equations – Cauchy's Problem for first order equations – Linear Equations of the first order – Nonlinear partial differential equations of the first order – Cauchy's method of characteristics – Compatible system of First order Equations – Solutions satisfying Given Condition, Jacobi's method

### **Unit-II: Partial Differential Equations of the Second order**

**Marks: 15**

The Origin of Second Order Equations – Linear partial Differential Equations with constant coefficients – Equations with variable coefficients – Separation of variables – The method of Integral Transforms – Non – linear equations of the second order.

### **Unit-III: Laplace's Equation**

**Marks: 15**

Elementary solutions of Laplace equation – Families of Equipotential Surfaces – Boundary value problems – Separation of variables – Surface Boundary Value Problems – Separation of Variables – Problems with Axial Symmetry – The Theory of Green's Function for Laplace Equation.

**Unit-IV: The wave equation****Marks: 15**

The Occurrence of the wave equation in Physics – Elementary Solutions of the One – dimensional Wave equations – Vibrating membrane, Application of the calculus of variations – Three-dimensional problem – General solutions of the Wave equation.

**Unit-V: The Diffusion Equation****Marks: 10**

Elementary Solutions of the Diffusion Equation – Separation of variables – The use of Integral Transforms – The use of Green's functions

**Text Book:**

- [1] Ian Sneddon – Elements of Partial Differential Equations – McGraw Hill International Book Company, New Delhi, 1983

**Reference Books:**

- [2] M.D. Raisinghania Advanced Differential Equations S. Chand and Company Ltd., New Delhi, 2001
- [3] K. Sankara Rao, Introduction to Partial Differential Equations, Second edition – Prentice – Hall of India, New Delhi 2006
- [4] J.N. Sharma & K. Singh Partial Differential Equations for Engineers & Scientists, Narosa Publishing House, 2001

**SEMESTER - I****Paper Code: MATADL 14044****Paper Title: Tensor Analysis****Theory Marks: 70****Internal Marks: 30****Course Outcome:**

By the end of this course, students will:

- Understand coordinate transformations, covariant and contravariant vectors, and tensor algebra.
- Work with symmetric, skew-symmetric, and mixed tensors.
- Use Christoffel symbols, solve geodesic equations, and perform covariant differentiation.
- Calculate vector divergence, curl, and the Laplacian operator.
- Learn about metric tensors, Riemannian spaces, and curvature tensors.
- Apply the Bianchi Identity and understand flat spacetime properties.
- Explore vector parallelism, tensor differentiation, and the Weyl tensor.

**Unit-I:****Marks: 20**

Curvilinear coordinates; Transformation of coordinates; Summation Convention; Dummy Suffix; Real Suffix; Covariant and Contravariant vectors; Tensors of Second Order; Mixed Tensors; Kronecker Delta; Algebra of Tensors; Symmetric and Skew-Symmetric tensors; Outer multiplication, Contraction and Inner Multiplication, Quotient Law of Tensors, Reciprocal Symmetric Tensor; Relative Tensor; Fundamental Tensor; Group property of tensors; Tensor Field.

**Unit-II:** **Marks: 20**  
Christoffel's symbols; Transformation of Christoffel's symbols; Differential equation of a Geodesic; Covariant differentiation of vectors; Covariant differentiation of tensors; Intrinsic derivative of a tensor; Laws of covariant differentiation of tensors; Divergence of a vector; Curl of a vector; Laplacian operator; Parallel displacement of a vector.

**Unit-III:** **Marks: 15**  
The metric tensor; Riemannian metric; Riemannian space; Geodesic coordinates; Natural coordinates; Riemannian Christoffel's tensor; Curvature of a curve; First curvature; Covariant curvature tensor; Properties of covariant curvature tensor; Bianchi Identity; Flat space time.

**Unit-IV:** **Marks: 15**  
Parallelism of vector of constant magnitude; Parallelism for vector of variable magnitude along a curve; Tensor differentiation; Laws of tensor differentiation; Weyl tensor.

**Reference Books:**

- [1] Tensor Calculus and Riemannian Geometry: D. C. Agarwal.
- [2] An Introduction to Riemannian Geometry and Tensor Calculus: Cambridge University Press: C. E. Weatherburn (1950).
- [3] Tensor Analysis: De Gruyter: Heinz Schade, Klaus Neemann, Andrea Dziubek, Edmond Rusjan (2018).

**SEMESTER - I**

**Paper Code: MATADL 14054**  
**Paper Title: Advanced Algebra**  
**Theory Marks: 70**  
**Internal Marks: 30**

**Course Outcome:** This course will state about Properties of external direct products, Group actions, stabilizers, Sylow 's theorems, Extension fields and splitting fields.

**Unit-I:** **Marks: 15**  
Properties of external direct products, the group of units modulo  $n$  as an external direct product, internal direct products, Fundamental Theorem of finite abelian groups.

**Unit-II:** **Marks: 20**  
Group actions, stabilizers and kernels, permutation representation associated with a given group action, Applications of group actions: Generalized Cayley's theorem, Index theorem.

**Unit-III:** **Marks: 20**  
Groups acting on themselves by conjugation, class equation and consequences, conjugacy in  $S_n$ ,  $p$ -groups, Sylow's theorems and consequences, Applications of Sylow's Theorems, Cauchy's theorem, Simplicity of  $A_n$  for  $n = 5$ , non-simplicity tests.

**Unit-IV:** **Marks: 15**  
Extension fields, splitting fields, zeros of an irreducible polynomial, perfect field, algebraic extension, finite fields, Construction of fields, Prime field.



**Reference Books:**

- [1] Modern Algebra by Surjeet Singh and Qasi Zameeruddin, Vikas Publishing House (Second Edition), New Delhi, 1975.
- [2] A First Course in Abstract Algebra by John B. Fraleigh, Published by Pearson Education (Singapore) Pte. Ltd.
- [3] Topics in Algebra, Second edition (Wiley Eastern Ltd.) by I.N. Herstein.
- [4] Joseph A. Gallian, Contemporary Abstract Algebra (Fourth Ed.), Narosa, 1999.
- [5] P. B. Bhattacharya, S. K. Jain and S. R. Nagpaul, Basic Abstract Algebra (Second Ed.), Cambridge Univ. Press (Indian Ed. 1995).

**SEMESTER - II****Paper Code: MATADL 14064****Paper Title: Mathematical Methods****Theory Marks: 70****Internal Marks: 30****Course Learning Objectives:**

1. Develop a comprehensive understanding of linear integral equations of the first and second kinds, including Fredholm and Volterra types, and learn methods to solve these equations using separable kernels.
2. Gain proficiency in the use of Laplace transforms, including their basic properties, convolution theorem, and inverse transforms, and apply these techniques to solve ordinary and partial differential equations in initial and boundary value problems.
3. Learn the properties of Fourier transforms, including Fourier sine and cosine transforms, and apply these techniques to solve ordinary and partial differential equations, as well as to evaluate definite integrals.
4. Understand the principles of calculus of variations, including the variation of a functional and the Euler-Lagrange equation, and apply these concepts to find necessary and sufficient conditions for extrema.
5. Develop methods to solve boundary value problems in ordinary and partial differential equations using variational methods and integral transforms.

**Course Learning Outcomes:**

1. Demonstrate the ability to solve linear integral equations of the Fredholm and Volterra types using methods involving separable kernels, and understand characteristic numbers, eigenfunctions, and the resolvent kernel.
2. Effectively use Laplace transforms and their properties to solve ordinary and partial differential equations, particularly in the context of initial and boundary value problems.
3. Apply Fourier transforms, including Fourier sine and cosine transforms, to solve ordinary and partial differential equations, and evaluate definite integrals using these techniques.
4. Utilize the principles of calculus of variations to derive and solve the Euler-Lagrange equation, and determine the necessary and sufficient conditions for extrema in various functional forms.
5. Solve boundary value problems in ordinary and partial differential equations using variational methods and integral transform techniques, demonstrating a comprehensive understanding of the mathematical methods involved.

**Unit-I:** **Marks: 20**

**Linear Integral Equations:** Linear integral equation of the first and second kind of Fredholm and Volterra type, Solutions with separable kernels. Characteristic numbers and eigen functions, Resolvent kernel.

**Unit-II:** **Marks: 20**

**Laplace Transform:** Basic properties of Laplace transform convolution theorem and properties of convolution, Inverse Laplace transform, Application of Laplace transform to the solution of ordinary and partial differential equations of initial and boundary value problems.

**Unit-III:** **Marks: 20**

**Fourier Transform:** Fourier integral transform properties of Fourier transform; Fourier sine and cosine transform application of Fourier transform to ordinary and partial differential equations initial and boundary value problems evaluation of the definite integral.

**Unit-IV:** **Marks: 10**

**Calculus of Variations:** Variation of a functional, Euler-Lagrange equation, Necessary and sufficient conditions for extrema. Variational methods for boundary value problems in ordinary and partial differential equations.

**Reference Books:**

- [1] M. D. Rai Singhania, Advanced Differential equations.
- [2] M. R. Spiegel Willard, Theory and Problems of Laplace Transform.
- [3] A. S. Gupta., Calculus of variation with Applications.
- [4] P. Jones, Python: The Fundamentals of Python Programming, CreateSpace Independent Pub.
- [5] S. Linge, H.P. Langtangen, Programming for computations, Springer.

**SEMESTER - II**

**Paper Code: MATSPL 15074 (A)**

**Paper Title: Analysis-II**

**Theory Marks: 70**

**Internal Marks: 30**

**Course Outcome:** Study about the properties of Riemann integral in more general framework by defining the integral as the limit of a sequence of simpler functions. Study about the convergence structure in LP Space.

**Unit-I:** **Marks: 15**

Lebesgue outer measure, Borel sets and their measurability, Characterisation of Measurable sets and non-measurable sets.

**Unit-II:** **Marks: 15**

Measurable functions, Littlewood's principles, Simple and step functions, Lebesgue integral of bounded functions, Bounded convergence theorem.

**Unit-III:****Marks: 20**

Integral of non-negative functions, Fatou's Lemma, Monotone convergence theorem, Comparison with Riemann integral, Lebesgue general integral, Lebesgue Dominated convergence theorem, countability additivity and continuity of integration, uniform integrability: Vitali convergence theorem, Convergence in measure, integration of convex functions.

**Unit-IV:****Marks: 20**

$L^p$  Space: Completeness and approximation, duality and weak convergence.

**Textbooks:**

- [1] Royden, H. L., Fitzpatrick, P. M., Real Analysis, 4<sup>th</sup> Edition, PHI Learning Private Limited, New Delhi, 2011
- [2] Barra, G. de, Measure Theory and Integration; Wiley-Eastern, 1981.

**Reference Books:**

- [1] Jain, P. K. and Gupta, V. P., Lebesgue Measure and Integration, New Age International(P) Limited, New Delhi, 1986.
- [2] Halmos, P. R., Measure Theory; Springer-Verlag, 1974.

## **SEMESTER - II**

**Paper Code: MATSPL 15074 (B)**

**Paper Title: Mathematical Modelling**

**External Marks: 70**

**Internal Marks: 30**

**Course Learning Objectives:**

1. Develop a comprehensive understanding of the definition, importance, and classification of mathematical modelling, and learn the process of creating elementary mathematical models.
2. Learn to model single-species population dynamics using exponential and logistic growth models, including harvesting models and determining their critical values.
3. Gain proficiency in modelling with ordinary differential equations, including concepts of stability, steady states, and applications in various fields such as economics, ecology, and epidemiology.
4. Learn to construct and analyze mathematical models using difference equations, including applications in economics, finance, and population dynamics, and understand the basic theory of linear difference equations with constant coefficients.
5. Understand the derivation and application of partial differential equations in various situations, including solving the one-dimensional heat equation and wave equation.

**Course Learning Outcomes:**

1. Demonstrate the ability to define, classify, and construct elementary mathematical models, and understand the role of mathematics in solving real-world problems.
2. Effectively model single-species population dynamics using exponential and logistic growth models, analyze harvesting models, and determine critical values.

3. Apply ordinary differential equations to model growth and decay processes, analyze the stability of solutions, and use these models in practical applications such as economics, ecology, and epidemiology, including understanding basic reproduction numbers.
4. Construct and analyze mathematical models using difference equations, solve linear difference equations with constant coefficients, and apply these models to problems in economics, finance, and population dynamics.
5. Derive and solve partial differential equations arising from various situations, specifically solving the one-dimensional heat equation and wave equation, and apply these solutions to practical modelling scenarios.

**Unit-I:**

**Marks: 20**

**Introduction to Mathematical Modelling:** Definition and Importance of Mathematical Modelling, Need, Classification, Modelling Process, Elementary Mathematical Models; Role of mathematics in Problem Solving.

**Single Species Population Models:** The Exponential Growth Model and the Logistic Growth Model, Harvesting Model and its Critical Value.

**Unit-II:**

**Marks: 20**

**Modelling with Ordinary Differential Equations:** Overview of Basic Concepts in ODE and Stability of Solutions, Steady State and their Local and Global Stability, Linear and Non-linear Growth and Decay Models. Compartment models. Some Applications in Economics, Ecology, Modelling in Epidemiology (SIS, SIR, SIRS Models) and Basic Reproduction Number.

**Unit-III:**

**Marks: 20**

Mathematical Models through Difference Equations, Some Simple Models, Basic theory of Linear Difference Equations with Constant Coefficients, Mathematical Modelling through Difference Equations in Economics and Finance, Mathematical Modelling through Difference Equations in Population Dynamics.

**Unit-IV:**

**Marks: 10**

Mathematical Modelling through Partial Differential Equations, Situations Giving Rise to of Partial Differential Equation Models. The One-Dimensional Heat Equation: Derivation and Solution. Wave Equation: Derivation and Solution.

**Reference Books:**

- [1] D. N. P. Murthy, N. W. Page and E. Y. Rodin, Mathematical Modelling, Pergamon Press.
- [2] J. N. Kapoor, Mathematical Modelling, Wiley Estern Ltd.
- [3] J.N. Kapur, Mathematical Models in Biology and Medicine, East-West Press.
- [4] F. Charlton, Ordinary Differential and Differential equation, Van Nostarnd.
- [5] Fred Brauer and Carlos Castillo-Chavez, Mathematical Models in Population Biology and Epidemiology, Springer.
- [6] Frank R. Giordano, William Price Fox, Maurice D. Weir, A First Course in Mathematical Modelling, 4th Ed., Charlie Van Wagner.

## **SEMESTER - II**

**Paper Code: MATSPL 15084 (A)**

**Paper Title: Graph Theory I**

**External Marks: 70**

**Internal Mark: 30**

**Course Outcome:** This course is an introduction to the theory of graphs intended for students of mathematics and other sciences. The course introduces in an elementary way some basic knowledge and primary methods in Graph Theory.

### **Unit-I:**

**Marks: 30**

The Basic: Graphs, Graph isomorphism, Regular and complete graph, Bipartite graph, Intersection Graph, line graph, block graph, Walk, trail, path, cycle, digraph, weighted graph and Adjacency matrix of a labeled graph, Degree partition.

### **Unit-II:**

**Marks: 20**

Tree: Basic properties of trees, Distance in trees, Spanning tree, shortest path. Connectivity: Cut vertex, cut edge, blocks, Point-connectivity (connectivity) and line connectivity

### **Unit-III:**

**Marks:20**

**Traversability:** Euler tour, Eulerian graph, The Chinese postman problem, Hamilton cycle, Hamiltonian graph, Travelling salesman problem. Covering: Point cover, Edge cover, independent sets, König theorems.

### **Books:**

- [1] Graph Theory, F. Harary, Narosha Publishing Co (Reprint 1998).
- [2] Introduction to Graph Theory, D. B. West, Prentice-Hall, (2001).
- [3] Basic Graph Theory, K. R. Parthasarathy, Tata McGraw Hill, (1994)
- [4] Graph Theory with Applications, J. A. Bondy & U. S. R. Murty, North-Holland (1976).

## **SEMESTER - II**

**Paper Code: MATSPL 15084 (B)**

**Paper Title: Continuum Mechanics**

**External Marks: 70**

**Internal Marks: 30**

### **Course Learning Objectives:**

1. Understand the basic principles and assumptions of continuum mechanics, including the continuum hypothesis, mass, density, and various types of forces and stresses.
2. Develop the ability to analyze and compute stress and strain in materials, including the use of stress and strain tensors, principal stresses and strains, and stress invariants.
3. Gain insight into the kinematics of deformation, including the description of motion using Lagrangian and Eulerian frameworks, and the calculation of deformation gradients and strain tensors.

4. Learn the principles of fluid mechanics, including the conservation laws of mass, momentum, and energy, and apply these principles to the study of both viscous and inviscid fluids.
5. Utilize mathematical tools such as tensors, differential equations, and compatibility conditions to solve complex problems in continuum mechanics, including those involving stress, strain, and fluid flow.

**Course Learning Outcomes:**

1. Demonstrate an understanding of the continuum hypothesis and calculate mass, density, body forces, and surface forces in a given material.
2. Calculate stress components using Cauchy's law and stress tensor, and interpret normal and shear stresses, principal stress, stress invariants, and stress deviators.
3. Accurately describe the deformation of materials using Lagrangian and Eulerian descriptions, material and spatial coordinates, and calculate displacement and deformation gradients.
4. Analyze steady, uniform, linear, and irrotational motion, and understand the concepts of path lines, streamlines, and vortex lines, applying Reynolds transport theorem to fluid motion problems.
5. Apply the conservation principles of mass, linear momentum, angular momentum, and energy to solve problems involving both incompressible and compressible fluids, and calculate fluid pressure and viscous stress tensors.

**Unit-I:**

**Marks: 20**

Continuum hypothesis, mass and density, body force and surface force, stress components, Cauchy's law, state of stress at a point, stress tensor, normal and shear stresses, principal stress, stress invariants, stress deviator, boundary condition for stress tensor.

**Unit-II:**

**Marks: 20**

Continuum configuration, Lagrangian and Eulerian description, material and spatial coordinates, deformation, displacement and deformation gradients, stretch and rotation tensor, strain tensor, strain-displacement relations, infinitesimal strain tensor, interpretation of linear strain tensor, compatibility conditions, principal strains, strain deviator.

**Unit-III:**

**Marks: 15**

Material and local time derivatives, velocity and acceleration, steady, uniform and linear motion, irrotational motion and potential flow, path lines, streamlines and vortex lines, Reynolds transport theorem, circulation and vorticity,

**Unit-IV:**

**Marks: 15**

Conservation of mass, continuity equation, linear momentum principle, equation of motion, angular momentum principle, general solution of the equation of equilibrium, energy equation. Viscous and inviscid fluids, viscous stress tensor, fluid pressure, incompressible and compressible fluids.

**Reference Books:**

- [1] Continuum Mechanics by D. S. Chandra Sekharaiah and Lokenath Debnath, Prism Books Pvt. Ltd., Bangalore.
- [2] Mathematical Theory of Continuum Mechanics by Rabindranath Chatterjee, Narosa Publishing House.
- [3] Schaum's Outline of Theory and Problems of Continuum Mechanics by George E. Mase., Schaum's Outline Series, McGraw-Hill.

## **SEMESTER - II**

**Paper Code: MATSPL 15094 (A)**

**Paper Title: General Topology**

**Theory Marks: 70**

**Internal Marks: 30**

**Course Outcome:** Able to study about the properties of Metric spaces mainly related to continuous functions, in more general framework known as topological spaces.

### **Unit-I:**

**Marks: 15**

Topological spaces, base and sub-base, subspaces. Closure, interior and boundary of a subset: their properties. Neighbourhood structures. Characterisation of topology in terms of closure and interior operator. Continuity, open and closed functions, homeomorphisms, strong and weak topologies. Quotient and product spaces (finite product).

### **Unit-II:**

**Marks: 20**

Countability axioms, separability, Lindelof spaces. Separation axioms (T<sub>0</sub>, T<sub>1</sub>, T<sub>2</sub>, T<sub>3</sub>, T<sub>4</sub>), regularity, complete regularity, normality, Urysohn's Lemma (Statement only) and its applications.

### **Unit-III:**

**Marks: 20**

Compactness, local compactness, compactification, The Stone-Cech compactification. Alexandroff one point compactification, Connectedness, components, local and path connectedness.

### **Unit-IV:**

**Marks: 15**

Tychonoff product (Product topology on arbitrary product), Separation axioms and product spaces, Compactness and product spaces, Connectedness and product spaces.

### **Textbooks:**

- [1] Willard, S. General Topology, Addison-Wesley, Reading, 1970
- [2] Munkres, J. R., Topology: A first course (2/e), Prentice-Hall, 2000

### **Reference Books:**

- [1] Joshi, K. D., Topology, Wiley-Eastern, 1988
- [2] Dugundji, J., Topology, Allyn and Bacon, 1966
- [3] Steen, L., Seebach, J., Counter Examples in Topology, Holt, Reinhart and Winston, New York, 1970

## **SEMESTER - II**

**Paper Code: MATSPL 15094 (B)**

**Paper Title: Differential Geometry**

**External Marks: 70**

**Internal Marks: 30**

**Course Learning Objectives:**

1. Gain a comprehensive understanding of space curves, including their fundamental properties such as curvature, torsion, and their related intrinsic equations.
2. Learn about the intrinsic properties of surfaces, including metrics, direction coefficients, and the intrinsic nature of various surfaces like surfaces of revolution and helicoids.
3. Develop a deep understanding of geodesics on surfaces, including their canonical equations, normal properties, and theorems related to geodesic parallels and curvatures.
4. Examine non-intrinsic properties of surfaces such as the second fundamental form, principal curvatures, lines of curvature, and the characteristics of developable and minimal surfaces.
5. Apply fundamental equations of surface theory to analyze and understand the existence and properties of compact and complete surfaces, and explore specific concepts like Hilbert's lemma and umbilic points.

### **Course Learning Outcomes:**

1. Demonstrate the ability to model space curves, calculate arc lengths, tangents, normals, binormals, and understand curvature, torsion, and the contact between curves and surfaces.
2. Characterize surfaces through their intrinsic properties, including isometric correspondence, metric properties, and the behavior of curves on these surfaces.
3. Solve problems involving geodesics on surfaces, including using canonical equations and understanding geodesic curvature, Gaussian curvature, and applying the Gauss-Bonnet Theorem.
4. Analyze and determine non-intrinsic properties of surfaces, such as principal curvatures and lines of curvature, and understand the properties of developable and minimal surfaces.
5. Apply the principles of surface theory to solve problems involving compact surfaces, surfaces of constant curvature, and complete surfaces, leveraging the fundamental existence theorems and specific geometric lemmas.

### **Unit-I:**

**Marks:20**

**SPACE CURVES:** Definition of a space curve - Arc length - tangent - normal and binormal - curvature and torsion - contact between curves and surfaces - tangent surface - involutes and evolutes - Intrinsic equations - Fundamental Existence Theorem for space curves - Helics.

### **Unit-II:**

**Marks: 20**

**INTRINSIC PROPERTIES OF A SURFACE:** Definition of a surface - curves on a surface - Surface of revolution - Helicoids - Metric - Direction coefficients - families of curves - Isometric correspondence - Intrinsic properties.

### **Unit-III:**

**Marks: 10**

**GEODESICS:** Geodesics - Canonical geodesic equations - Normal property of geodesics - Existence Theorems - Geodesic parallels - Geodesics curvature - Gauss - Bonnet Theorem - Gaussian curvature - surface of constant curvature.

### **Unit-IV:**

**Marks: 10**

**NON-INTRINSIC PROPERTIES OF A SURFACE:** The second fundamental form - Principal curvature - Lines of curvature - Developable - Developable associated with space curves and with curves on surface - Minimal surfaces - Ruled surfaces.



**Unit-V:****Marks: 10**

**DIFFERENTIAL GEOMETRY OF SURFACES:** Fundamental Equations of Surface Theory - Fundamental Existence Theorem for surfaces - Compact surfaces whose points are umbilics - Hilbert's lemma - Compact surface of constant curvature - Complete surfaces.

**Reference Books**

- [1] T. J. Willmore, An Introduction to Differential Geometry, Oxford University Press, (17th Impression) New Delhi 2002. (Indian Print)
- [2] Struik, D.T. Lectures on Classical Differential Geometry, Addison - Wesley, Mass. 1950.
- [3] Kobayashi. S. and Nomizu. K. Foundations of Differential Geometry, Interscience Publishers, 1963.
- [4] Wilhelm Klingenberg: A course in Differential Geometry, Graduate Texts in Mathematics, Springer-Verlag 1978.
- [5] 4. J.A. Thorpe Elementary topics in Differential Geometry, Under - graduate Texts in Mathematics, Springer - Verlag 1979.

**SEMESTER - II****Paper Code: MATSPL 15104****Paper Title: Computer Lab I****Practical Marks: 70****Internal Marks: 30****Unit-I: Introduction to Numerical Computation**

- Overview of Numerical Computation
- Sources of Error: Round-off Error, Truncation Error
- Floating-Point Arithmetic
- Error Analysis and Stability
- Taylor Series Approximation
- Introduction to MATLAB/Python for Numerical Computation

**Unit-II: Solving Equations and Optimization**

- Root Finding: Bisection Method, Newton-Raphson Method, Secant Method
- Systems of Linear Equations: Gaussian Elimination, LU Decomposition
- Interpolation and Approximation: Lagrange Interpolation, Polynomial Approximation
- Numerical Differentiation and Integration
- Optimization Techniques: Gradient Descent, Newton's Method

**Unit-III: Numerical Linear Algebra**

- Matrix Operations: Addition, Multiplication, Inversion
- Eigenvalue and Eigenvector Computation: Power Method, QR Algorithm
- Least Squares Approximation
- Applications in Image Processing, Graph Theory, Signal Processing, and Data Analysis

**Unit-IV: Numerical Solution of Differential Equations**

- Ordinary Differential Equations (ODEs): Euler Method, Runge-Kutta Methods
- Stability and Convergence Analysis

- Boundary Value Problems: Shooting Method, Finite Difference Method
- Applications in Computational Biology, Fluid Dynamics

**Textbook:**

- [1] P. Jones, Python: The Fundamentals of Python Programming, CreateSpaceIndependent Pub.
- [2] S. Linge, H. P. Langtangen, Programming for computations, Springer.
- [3] E. V. Krishnamurthy and S. K. Sen: Computer Based Numerical Algorithms, East-West press Pvt. Ltd. 1976.
- [4] R. Johansson, Numerical Python, A Press.
- [5] Numerical Methods for Engineers by Steven C. Chapra and Raymond P. Canale

**References:**

- [1] Numerical Recipes: The Art of Scientific Computing by William H. Press et al.
- [2] Introduction to Scientific Computing: A Matrix-Vector Approach Using MATLAB by Charles F. Van Loan

**SEMESTER - III**

**Paper Code: MATSPL 25014 (A)**

**Paper Title: Fuzzy Set Theory**

**Theory Marks: 70**

**Internal Marks: 30**

**Course Outcomes:** This course will help the students to learn about the fuzzy set theory and recent practical development of fuzzy set theory in the present environment.

**Unit-I:**

**Marks: 30**

Fuzzy sets - basic definitions,  $\alpha$ -level sets, convex fuzzy sets, basic operations on fuzzy sets, types of fuzzy sets, Cartesian products, algebraic products, bounded sum and difference, t-norms and t-conforms. Fuzzy sets in contrast of probability theory.

**Unit-II:**

**Marks: 20**

The extension principle - the Zadeh's extension principle, image and inverse image of fuzzy Sets, fuzzy numbers, elements of fuzzy arithmetic.

**Unit-III:**

**Marks: 20**

Fuzzy relations and fuzzy graphs, composition of fuzzy relations, min-max composition and its properties, fuzzy equivalence relations, fuzzy relation equations, fuzzy graphs.

**Reference Books:**

- [1] Zimmermann, H. J., Fuzzy set theory and its Applications, Allied publishers Ltd., New Delhi, 1991.
- [2] Klir, G. J. and Yuan, B., Fuzzy Sets and Fuzzy Logic: Theory and Applications, Prentice Hall of India, New Delhi, 1997.

## SEMESTER - III

**Paper Code: MATSPL 25014 (B)**

**Paper Title: Fluid Dynamics I**

**Theory Marks: 70**

**Internal Marks: 30**

### **Course Learning Objectives:**

1. Comprehend the fundamental properties and dynamics of real and ideal fluids, including velocity, acceleration, and continuity equations.
2. Learn to derive and apply Bernoulli's theorem in various fluid flow scenarios, understanding its implications for fluid motion and energy conservation.
3. Develop skills to analyze two-dimensional fluid motion using complex velocity potentials, conformal mapping, and specific flow patterns around objects.
4. Gain insights into the stress components in real fluids, the relationship between stress and velocity gradients, and perform stress analysis in fluid motion.
5. Study vortex motion, including circular and rectilinear vortices, and understand the concepts of boundary layer theory, including boundary layer thickness and associated properties.

### **Course Learning Outcomes:**

1. Demonstrate the ability to model and analyze fluid flow, using concepts such as velocity potential, streamlines, path lines, and streak lines, and apply Lagrange's and Euler's equations of motion.
2. Solve problems involving two-dimensional fluid motion using complex velocity potentials, sources, sinks, doublets, and conformal mapping techniques.
3. Conduct stress analysis in fluid motion, relating stress components to velocity gradients and understanding the rate of strain quadric of fluid elements.
4. Analyze elementary properties of vortex motion, apply Kirchhoff's vortex theorem, and solve problems involving motions due to circular and rectilinear vortices.
5. Apply boundary layer theory concepts to practical problems, calculate boundary layer thickness, displacement, momentum, and energy thickness, and understand their implications for fluid flow near boundaries.

### **Unit-I:**

**Marks: 20**

Real fluids and Ideal fluids; Velocity and Acceleration of a fluid particle; Equation of continuity; Boundary Surface; Stream lines, Path lines and streak lines; Velocity potential; Equation of motion: Lagrange's and Euler's equations of motion; Bernoulli's Theorem.

### **Unit-II:**

**Marks: 20**

Motion in two-dimensions; Complex velocity potential; Sources, sinks, doublets and their images; Conformal mapping; Milne-Thomson circle theorem; Two-dimensional irrotational motion produced by motion of circular, co-axial and elliptic cylinders in an infinite mass of liquid; Theorem of Blasius; Stoke's stream function.

### **Unit-III:**

**Marks: 15**

Stress components in a real fluid; Relations between rectangular components of stress. Connection between stresses and gradients of velocity, Stress analysis in fluid motion, Rate of strain quadric of fluid element, Transformation of rate of strain components.

**Unit-IV:****Marks: 15**

Vortex motion and its elementary properties; Motions due to circular and rectilinear vortices; Kirchhoff vortex theorem. Boundary layer theory: Boundary layer concept, boundary layer thickness, displacement, momentum, and energy thickness.

**Reference Books:**

- [1] Fluid Dynamics - William F. Hughes, John A. Brighton, Schaum's Outline Series, McGraw-Hill.
- [2] Boundary-Layer Theory - Hermann Schlichting, Klaus Gersten, Springer.
- [3] Chorlton, Text Book of Fluid Dynamics, CBS Publishers, Delhi, 1985.
- [4] G. K. Batchelor, An Introduction to Fluid Mechanics, Foundation Books, New Delhi, 1994.
- [5] H. Schlichting, Boundary Layer Theory, McGraw Hill Book Company, New York, 1971.
- [6] M. D. Rai Singhanian, Fluid Mechanics (With Hydrodynamics) S. Chand and Company Ltd., New Delhi.
- [7] R. K. Rathy, An Introduction to Fluid Dynamics, Oxford and IBH Publishing Company, New Delhi, 1976.

**SEMESTER - III****Paper Code: MATSPL 25014 (C)****Paper Title: Advanced Topology I****Theory Marks: 70****Internal Marks: 30**

**Course Outcome:** Study about the properties of Metric spaces related to convergence structure in topological spaces. Metrization based on paracompactness. The role of fundamental group in the study of topological spaces.

**Unit-I:****Marks: 20**

Nets and filters, convergence in terms of nets and filters, ultrafilters and compactness. Theories of metrization, Urysohn's Lemma, Tietze Extension theorem, Urysohn metrization Theorem.

**Unit-II:****Marks: 15**

Paracompactness, characterisation in regular spaces, metrization based on paracompactness, Nagata-Smirnov theorem, Stone's theorem, Smirnov's metrization theorem.

**Unit-III:****Marks: 20**

Topological groups, subgroups, quotient groups, homogeneous spaces, product groups.

**Unit-IV:****Marks: 15**

Homotopy and the fundamental group, computation of the fundamental group of the circle.

**Text Books:**

- [1] Willard, S. General Topology, Addison-Wesley, Reading, 1970
- [2] Munkres, J. R., Topology: A first course (2/e), Prentice-Hall, 2000

**Reference Books:**

- [1] Joshi, K. D., Topology, Wiley-Eastern, 1988.
- [2] Kelley, J. L., Topology, Van Nostrand, 1955.

**SEMESTER - III****Paper Code: MATSPL 25024 (A)****Paper Title: Graph Theory II****Theory Marks: 70****Internal Marks: 30**

**Course Outcome:** This course is about to the topics of graph theory like Factorization, planarity, colourability matching and domination. The course introduces basic graph algorithms with applications.

**Unit-I:****Marks:15**

Factorization: factors, 1-factorization, 2-factorization, Tutte's 1-factor theorem. Planarity: Graph embeddings; Planar graph, Euler's formula, Parameters of planarity.

**Unit-II:****Marks: 20**

Colourability: Vertex and edge colouring, Chromatic number and edge chromatic number, Bounds for chromatic number, Brooks' theorem; Vizing's theorem, The four colour theorem, The five colour conjecture, Uniquely colourable graph.

**Unit-III:****Marks: 25**

Matching: Basic concepts of matching, maximum, maximal and perfect matching, Augmented path, Berge's theorem, Matching in bipartite graph, König's theorem for maximum matching. Domination: Dominating set and dominating number, Closed and open neighbourhood, Total dominating set and total domination number.

**Unit-IV:****Marks:10**

Basic Graph Algorithms: Traversal algorithm like breadth first search, depth first search, Designing and analyzing of algorithms, Topological sort.

**Books:**

- [1] Introduction to Graph Theory, D. B. West, Prentice-Hall, (2001).
- [2] Basic Graph Theory, K. R. Parthasarathy, Tata McGraw Hill, (1994)
- [3] Graph Theory, F. Harary, Narosha Publishing Co (Reprint 1998).
- [4] Algorithm Design: J. Kleinberg and E. Tardos (2006) Pearson Education.
- [5] Introduction to Algorithms: H. Cormen, C.E. Leiserson and R.L. Rivest (2009) MIT Press.

## SEMESTER - III

**Paper Code: MATSPL 25024 (B)**

**Paper Title: Special Theory of Relativity**

**Theory Marks: 70**

**Internal Marks: 30**

### **Course Learning Objectives:**

1. Develop an understanding of inertial and non-inertial frames, the geometry of Newtonian mechanics, and the transformations between these frames, particularly Galilean transformations.
2. Comprehend the fundamental postulates of the special theory of relativity, including Lorentz transformations and their implications for physical phenomena such as length contraction, time dilation, and simultaneity.
3. Study the variation of mass with velocity, the equivalence of mass and energy, and the transformations of mass, force, acceleration, density, momentum, and energy in relativistic contexts, including the formulation of relativistic Lagrangian and Hamiltonian mechanics.
4. Gain insights into Minkowski space and its role in representing relativistic concepts geometrically, including four-vectors, four-velocity, four-force, and four-momentum, and the covariant four-dimensional formulation of the laws of mechanics.
5. Learn the fundamentals of electrodynamics within the framework of special relativity, including the transformation of differential operators, Maxwell's equations, electromagnetic waves, and the transformation properties of electric and magnetic fields.

### **Course Learning Outcomes:**

1. Demonstrate the ability to apply Galilean and Lorentz transformations to solve problems in Newtonian mechanics and special relativity, including calculating length contraction, time dilation, and relativistic velocity addition.
2. Solve problems involving the variation of mass with velocity, the equivalence of mass and energy, and transform physical quantities such as mass, force, acceleration, density, momentum, and energy using relativistic principles.
3. Use Minkowski space to represent and analyze relativistic phenomena, including constructing and interpreting four-vectors, and solving relativistic equations of motion using covariant four-dimensional formulations.
4. Apply the principles of electrodynamics within the context of special relativity to transform differential operators, solve Maxwell's equations, analyze electromagnetic waves, and understand the transformations of electric and magnetic field components.
5. Integrate concepts from special relativity and electrodynamics to solve complex problems, demonstrating a thorough understanding of the interplay between relativistic mechanics and electromagnetic theory.

### **Unit-I:**

**Marks: 20**

Inertial and non-inertial frames, Geometry of Newtonian mechanics, Galilean Transformations, Fundamental postulates of the special theory of relativity, Lorentz transformation, Lorentz transformation as a group. Length contraction, Time dilation and Simultaneity. Relativistic addition law of velocities.

**Unit-II:****Marks: 15**

Relativistic mechanics, Variation of mass with velocity, Equivalence of mass and energy. Transformation of mass, force acceleration, density, momentum and energy. Relativistic Lagrangian and Hamiltonian.

**Unit-III:****Marks: 15**

Minkowski's space, Geometrical representation of simultaneity, contraction and time dilation, Space-like and time-like intervals, Position four vectors, four-velocity, four forces and four momentums, Relativistic equations of motion. Covariant four-dimensional formulations of laws of mechanics.

**Unit-IV:****Marks: 20**

**Electrodynamics:** Fundamental of electrodynamics, Transformation of differential operators, D'Alembert operator, Maxwell's equations and electromagnetic waves, Electromagnetic potentials and force Lorentz condition, Transformations of charge and current density, transformation equations of electric field strength and magnetic field induction components.

**Reference Books:**

- [1] Introduction to Special Relativity, Wiley Eastern Ltd. (1990) Robert.
- [2] The Mathematical Theory of Relativity, Cambridge University Press 1965 A. S. Eddington.
- [3] Relativity, Thermodynamics and Cosmology - R.C. Tolman (Oxford Press)

**SEMESTER - III****Paper Code: MATSPL 25024 (C)****Paper Title: Category Theory I****Theory Marks: 70****Internal Marks: 30**

**Course Outcome:** Study about the fundamental structure and basic properties of Category Theory as Categories are algebraic structures with many complementary natures, e.g., geometric, logical, computational, combinatorial, just as groups are many-faceted algebraic structures.

**Unit-I:****Marks:15**

Definition and examples of categories, The concept of functor and the category  $\text{Cat}$ . Natural Transformations, isomorphism, epimorphism, monomorphism, Morphisms, Epis and Zeros Foundations, Large Categories, Hom-sets. Free Categories, Constructions of new categories from old categories, The Dual Category, The Arrow Category, The Slice and Co-Slice Category.

**Unit-II:****Marks: 20**

Epis and mono, Initial and Terminal objects, Generalized elements, Sections and Retractions, Categories of categories, properties of functors, natural transformation and natural isomorphism, isomorphisms and equivalences of categories, functor categories.

**Unit-III:** **Marks: 20**  
Groups in categories. The category of groups, Groups as categories, Congruence on a category, quotient category and its univalent mapping property, finitely presented categories.

**Unit-IV:** **Marks:15**  
Equalizers and coequalizers, intersections and factorizations, products and coproducts, sources and sinks, limits and colimits, pullback and pushout, inverse and direct limits, complete categories, limits in factor categories.

**Text Books:**

- [1] Awodey, S.: Category Theory, (Oxford Logic Guides, 49, Oxford University Press.)

**Reference Books:**

- [1] Herrlich, Horst; Strecker, George E. (2007), Category Theory (3rd ed.), Heldermann Verlag Berlin

### **SEMESTER - III**

**Paper Code: MATSPL 25034 (A)**

**Paper Title: Number Theory I**

**Theory Marks: 70**

**Internal Marks: 30**

**Course Outcome:** Basic study of arithmetic number theory including divisibility, congruences, linear Diophantine equation.

**Unit-I:** **Marks: 15**  
Divisibility, greatest common divisor, least common multiple, Euclidean Algorithm. Chinese Remainder Theorem. Prime numbers, factorization in prime numbers, fundamental theorem of arithmetic.

**Unit-II:** **Marks: 15**  
Divisor functions, perfect numbers, Mersenne numbers, Fermat numbers.

**Unit-III:** **Marks: 20**  
Greatest integer function (Gauss function), Mobius function, Euler function. Congruences and its elementary properties, congruences in one unknown, complete residue system, reduced residue system.

**Unit-IV:** **Marks: 20**  
Diophantine equations, linear Diophantine equations, Pythagoras equation, sum of two squares. Quadratic residues and congruences of second degree in one unknown, Legendre symbol, Jacobi symbol, congruences of second degree with prime modulus and with composite modulus.

**Text Books:**

- [1] Hardy, G.H. and Wright, E. M. An Introduction to the Theory of Numbers, 4th Edition (Oxford University Press, 1960).



[2] Andrews, G.E. Number Theory (Hindustan Publishing Corporation, New Delhi, 1992).

**Reference Books:**

- [1] Burton, D. M. Elementary Number Theory, 6th Edition (Tata McGraw-Hill, New Delhi, 2007).
- [2] Niven, I. and Zuckerman, H. An Introduction to the Theory of Numbers, 5th Edition (Wiley Eastern, New Delhi, 2000).

**SEMESTER - III**

**Paper Code: MATSPL 25034 (B)**

**Paper Title: Dynamical Systems I**

**Theory Marks: 70**

**Internal Marks: 30**

**Unit-I:**

**Marks: 20**

Dynamical systems, discrete and continuous dynamical systems, Examples of dynamical systems. Iteration, Orbits, Types of Orbits, Other Orbits, The Doubling Function.

**Unit-II:**

**Marks: 25**

Graphical Analysis, Orbit Analysis, The Phase Portrait, Examples, Solutions Fixed and Periodic Points, A Fixed-Point Theorem, Attraction and Repulsion. Stability of fixed points and equilibrium points.

**Unit-III:**

**Marks: 25**

Bifurcations: Dynamics of the Quadratic Maps, Saddle Node Bifurcation, Period Doubling Bifurcations, The Quadratic Family

**Reference Books:**

- [1] Differential Equation, Dynamical System and Linear Algebra: Morris W. Hirsch, Stephen Smale, Academic Press.
- [2] An introduction to Dynamical systems: D. K. Arrowsmith
- [3] A First Course in Chaotic Dynamical Systems: Robert L. Devaney

**SEMESTER - III**

**Paper Code: MATSPL 25044 (A)**

**Paper Title: Advanced Functional Analysis**

**Theory Marks: 70**

**Internal Marks: 30**

**Course Outcome:** This course is about the basic of Topological vector spaces, Banach and  $C^*$  algebras. The course deals with various properties of different types of linear operators. This course also included fixed point theory

**Unit-I:** **Marks: 20**  
Topological Vector Spaces: Introduction, Separation properties, Linear mappings, Finite-dimensional spaces, Metrization, Boundedness and continuity, Seminorms and local convexity, Quotient space, completeness, convexity, Weak topologies, Compact convex sets.

**Unit-II:** **Marks: 20**  
Preliminaries on Banach Algebras and C\* Algebras, Commutative Banach Algebras and Commutative C\* Algebras, Representation of C\* Algebras.

**Unit-III:** **Marks:20**  
Spectral Theory of linear operators in normed space, Spectral Properties of Bounded Linear Operators, Further Properties of Resolvent, Spectrum and Banach Algebras. Spectral Properties of Bounded Self-Adjoint Linear Operators, Positive Operators, Square Roots of a Positive Operator, Projection Operators, Spectral Family of a Bounded Self-Adjoint Linear, Spectral Representation of Bounded Self-Adjoint Linear Operators.

**Unit-IV:** **Marks: 10**  
Fixed Point Theorems and Some Applications to Analysis.

**Books:**

- [1] Kreyszig E., Introductory Functional Analysis with Applications (John Wiley and Sons, New York, 1978).
- [2] Rudin, W. Functional Analysis (McGraw Hill, 2000).
- [3] Simmons, G. F. Introduction to Topology and Modern Analysis (Tata McGraw Hill Book Co. Ltd.,1963).
- [4] Gerard J. Murphy, C\* - Algebras and Operator Theory, Academic Press, Inc, 1990.
- [5] Ronald G. Douglas, Banach Algebra Techniques in Operator Theory, Second Edition, Springer-Verlag, New York, Inc, 1998.
- [6] Limaye, B. V. Functional Analysis (Wiley Eastern Ltd., New Delhi, 1989).

### **SEMESTER - III**

**Paper Code: MATSPL 25044 (B)**  
**Paper Title: Numerical Analysis I**  
**Theory Marks: 70**  
**Internal Marks: 30**

**Course Outcome:**

By the end of this course, students will:

- Solve linear equations using LU and Cholesky decompositions, and iterative methods.
- Analyze eigenvalues and eigenvectors with methods like the power method and QR algorithm.
- Solve non-linear equations using Newton-Raphson and Steffensen's iteration.
- Perform numerical integration with Gauss-Legendre, Euler-Maclaurin, and Romberg techniques.

**Unit-I:****Marks: 20**

Direct method for solving of linear equations (Crout's method, LU decomposition, Cholesky decomposition), Iterative methods (Relaxation method), Escalator method Steepest descent and conjugate gradient method.

**Unit-II:****Marks: 20**

Algebraic Eigen value problem: Properties of Eigen values and Eigen vectors, Power method, Inverse power method, Jacobi's method, Given's method. Orthogonal factorization, QR algorithm for Eigen value problem Eigen values of complex matrix and complex Eigen vectors.

**Unit-III:****Marks: 15**

System of non-linear method equations: Solution of Non-linear Equations: Single Equation: Modified Newton-Raphson method (for real roots-simple or repeated). Aitken's  $\Delta^2$ -method and Steffensen's iteration. Bairstow's method of quadratic factors, Graeffe's root squaring method. Non-Linear Systems of Equations: Newton's method, Quasi-Newton's method.

**Unit-IV:****Marks: 15**

Numerical Integration: Gauss- Legendre and Gauss-Chebyshev Quadrature's, Euler-Maclaurin summation formula, Romberg integration.

**Reference Books:**

- [1] An introduction to Numerical Analysis: Kendal E. Atkinson, Johan Wiley and sons, Inc.
- [2] Numerical Methods in engineering & Science: Dr. B. S. Grewal
- [3] Introduction to Numerical analysis: C. E. Froberg, Addison Wesley publishing Company, sixth edition, 1981.
- [4] Introductory Methods of Numerical Analysis: S. S. Sastri, Prentice Hall of India, New Delhi, 1997.

**SEMESTER - III****Paper Code: MATSPL 25054****Paper Title: Computer Lab II****Practical Marks: 70****Internal Marks: 30****Unit-I: Introduction to Numerical Computation**

- Review of Numerical Computation Fundamentals
- Advanced Error Analysis
- Introduction to High-Performance Computing
- Parallel and Distributed Computing
- GPU Computing

**Unit-II: Solving Nonlinear Equations and Optimization**

- Multivariate Root Finding Methods
- Nonlinear Optimization Techniques
- Constrained Optimization
- Global Optimization Methods

### **Unit-III: Numerical Linear Algebra II**

- Singular Value Decomposition (SVD)
- Iterative Methods for Linear Systems
- Sparse Matrix Computations
- Applications in Machine Learning and Big Data Analytics

### **Unit-IV: Advanced Topics in Computational Mathematics**

- Numerical Methods for Partial Differential Equations (PDEs)
- Spectral Methods
- Finite Element Methods for PDEs
- Applications in Computational Fluid Dynamics, Solid Mechanics, and Electromagnetics.

### **Textbook:**

- [1] P. Jones, Python: The Fundamentals of Python Programming, CreateSpace Independent Pub.
- [2] S. Linge, H. P. Langtangen, Programming for computations, Springer.
- [3] E. V. Krishnamurthy and S. K. Sen: Computer Based Numerical Algorithms, East-West press Pvt. Ltd. 1976.
- [4] R. Johansson, Numerical Python, A Press.
- [5] Numerical Methods for Engineers by Steven C. Chapra and Raymond P. Canale

### **References:**

- [1] Numerical Recipes: The Art of Scientific Computing by William H. Press et al.
- [2] Introduction to Scientific Computing: A Matrix-Vector Approach Using MATLAB by Charles F. Van Loan.

## **SEMESTER - IV**

**Paper Code: MATSPL 25064 (A)**

**Paper Title: Fuzzy Logic and Control System**

**Theory Marks: 70**

**Internal Marks: 30**

**Course outcomes:** This course will be beneficial for the students to learn about the application of fuzzy logic and its application. Also, this course will give the idea about multi criteria decision making.

### **Unit-I:**

**Marks: 15**

Fuzzy logic, fuzzy propositions, fuzzy quantifiers, linguistic variables, inference from conditional fuzzy propositions, compositional rule of inference.

### **Unit-II:**

**Marks: 20**

Approximate reasoning - an overview of fuzzy expert systems, fuzzy implications and their selection, multi-conditional approximate reasoning, role of fuzzy relation equation.

### **Unit-III:**

**Marks: 15**

An introduction to fuzzy control - fuzzy controllers, fuzzy rule base, fuzzy inference engine, fuzzification, defuzzification and the various defuzzification methods.

**Unit-IV:****Marks: 20**

Decision making in fuzzy environment - individual decision making, multi-person decision making, multi-criteria decision making, multistage decision making, fuzzy ranking methods, fuzzy linear programming, fuzzy logic as a tool in soft computing.

**Reference Books:**

- [1] Zimmermann, H. J., Fuzzy set theory and its Applications, Allied publishers Ltd., New Delhi, 1991.
- [2] Klir, G. J. and Yuan, B., Fuzzy Sets and Fuzzy Logic: Theory and Applications, Prentice Hall of India, New Delhi, 1997.

**SEMESTER - IV**

**Paper Code: MATSPL 25064 (B)**  
**Paper Title: Numerical Analysis II**  
**Theory Marks: 70**  
**Internal Marks: 30**

**Course Outcome:**

By the end of this course, students will:

- Solve ordinary differential equations using methods like Modified Euler's and Adams-Bashforth.
- Address boundary-value problems with numerical techniques and the finite-difference method.
- Develop finite-difference approximations and solve elliptic equations for partial differential equations.
- Use relaxation methods and appropriate techniques for parabolic and hyperbolic equations.
- Apply different types of approximations, including least square polynomial and Chebyshev polynomials.

**Unit-I:****Marks: 15**

Numerical Solution of Ordinary Differential equations: Modified Euler's method, Predictor-corrector method, Milne's method, Adams-Bashforth method, Boundary-value problems, Finite-difference method.

**Unit-II:****Marks: 20**

Numerical Solution of Partial Differential equations: Finite-Difference approximations to partial derivatives, Elliptic equations, Solution of Laplace equation, Solution of Poisson's equation.

**Unit-III:****Marks: 20**

Solution of Elliptic Equations by Relaxation Method, Parabolic Equations, Hyperbolic Equations.

**Unit-IV:****Marks: 15**

Approximations: Different types of approximation, least square polynomial approximation, polynomial approximation by use of orthogonal polynomials, approximation with Chebyshev polynomials.

**Reference Books:**

- [1] Numerical Methods in Engineering & Science: Dr. B. S. Grewal
- [2] An introduction to Numerical Analysis: Kendal E. Atkinson, Johan Wiley and sons, Inc.
- [3] Introduction to Numerical analysis: C. E. Froberg, Addison Wesley publishing Company, sixth edition, 1981.
- [4] Introductory Methods of Numerical Analysis: S. S. Sastri, Prentice Hall of India, New Delhi, 1997.

**SEMESTER - IV**

**Paper Code: MATSPL 25064 (C)**  
**Paper Title: Advanced Topology II**  
**Theory Marks: 70**  
**Internal Marks: 30**

**Course Outcome:** Study about the notion related to the uniform structure in topological spaces. Introduction to the notion of Function space and Proximity structure.

**Unit-I:****Marks: 20**

Uniformities, uniform continuity, product uniformities, metrisation, completeness and Compactness, completion.

**Unit-II:****Marks: 20**

Uniform structures in Topological groups, complete groups, completion of topological groups.

**Unit-III:****Marks: 15**

Function spaces, pointwise convergence, uniform convergence, compact-open topology, k-spaces, equicontinuity, Ascoli theorem.

**Unit-IV:****Marks: 15**

Proximity Structure: Smirnov Compactification.

**Text Books:**

- [1] Willard, S. General Topology, Addison-Wesley, Reading, 1970
- [2] Munkres, J. R., Topology: A first course (2/e), Prentice-Hall, 2000

**Reference Books:**

- [1] Joshi, K. D. Topology, Wiley-Eastern, 1988.
- [2] Kelley, J. L. Topology, Van-Nostrand, 1955.
- [3] Bourbaki, N. Elements of Mathematics: General Topology, Vols. I & II, Springer-Verlag, 1988.

## **SEMESTER - IV**

**Paper Code: MATSPL 25074 (A)**

**Paper Title: Graph Theory III**

**Theory Marks: 70**

**Internal Marks: 30**

**Course Outcome:** This course is an introduction to the generalizations of graphs. The course introduces in an elementary way some basic knowledge of hypergraph and semigraph. This course also included some important algorithms.

**Unit-I: Marks: 20**

**Hypergraph:** Introduction to Hypergraph, Degree and rank, Incidence matrix, Cycles in hypergraph, conformal hypergraphs, Representative graph of a hypergraph, Dual of hypergraph.

**Unit-II: Marks: 25**

**Semigraph:** Introduction to Semigraph, Degrees in semigraph; Subsemigraph and partial subsemigraph, Path and cycle, Complete Semigraph, Strongly Complete Semigraph, Edge bipartite, Edge regular, Edge Complete Semigraph, Dendroids (Semitree).

**Unit-III: Marks: 25**

**Graph Algorithms:** Interval scheduling, Minimum spanning tree, Kruskal's algorithm, Prim's algorithm, Single-source shortest path, The Bellman-Ford algorithm, Dijkstra's algorithm, strongly connected components.

### **Books:**

- [1] Introduction to Graph Theory, D. B. West, Prentice-Hall, (2001).
- [2] Graphs and Hypergraphs, C. Berge, North-Holland, London (1973).
- [3] Hypergraphs, C. Berge, North-Holland, London (1973).
- [4] Semigraph and Their Application, E. Sampathkumarachar, Academy of Discrete Mathematics and Applications, Lecture Notes, Series No. 1 (2019).
- [5] Algorithm Design: J. Kleinberg and E. Tardos (2006) Pearson Education.
- [6] Introduction to Algorithms: H. Cormen, C.E. Leiserson and R.L. Rivest (2009) MIT Press

## **SEMESTER - IV**

**Paper Code: MATSPL 25074 (B)**

**Paper Title: Relativity and Cosmology**

**Theory Marks: 70**

**Internal Marks: 30**

### **Course Learning Objectives:**

1. Develop a comprehensive understanding of the foundational principles of general relativity, including the principles of equivalence, covariance, and geodesic principle.

2. Learn to derive and understand Einstein's field equations, including the Newtonian approximation, and analyze solutions such as the Schwarzschild external solution and its isotropic form.
3. Study and interpret various relativistic phenomena, such as redshift, the advance of perihelion of a planet, bending of light rays in a gravitational field, and the energy-momentum tensor of a perfect fluid.
4. Gain insights into static and non-static cosmological models, including the derivation and properties of the Einstein and De-Sitter models, and understand Hubble's law, cosmological principles, and the Robertson-Walker metric.
5. Understand the fundamental equations of dynamical cosmology, explore Friedmann models, and analyze concepts such as critical density, closed and open universes, the age of the universe, and steady state cosmology.

**Course Learning Outcomes:**

1. Demonstrate the ability to apply the principles of equivalence, covariance, and geodesic principle to solve problems in general relativity and understand their implications for the motion of objects in a gravitational field.
2. Successfully derive Einstein's field equations using variational principles and solve them for specific cases like the Schwarzschild external solution, understanding their physical significance in various relativistic scenarios.
3. Analyze and interpret the effects of general relativity on planetary orbits, the advance of perihelion, bending of light, and gravitational redshift, and understand their observational consequences.
4. Construct static and non-static cosmological models, derive and compare the properties of the Einstein and De-Sitter models with observations of the actual universe, and understand the implications of Hubble's law and the Robertson-Walker metric.
5. Apply the fundamental equations of dynamical cosmology to solve problems related to Friedmann models, calculate critical density, and determine the age and structure of the universe, understanding concepts like particle and event horizons and the steady state cosmological principle.

**Unit-I:**

**Marks: 20**

General Relativity, Principles of equivalence, Principle of covariance, Geodesic principle, Newtonian approximation of equation of motion, Einstein's field equations, Schwarzschild external solution and its isotropic form, Redshift, Tests of general theory of relativity, Planetary orbits and analogues of Kepler's Laws in general relativity.

**Unit-II:**

**Marks: 20**

Advance of perihelion of a planet, Bending of light rays in a gravitational field, Gravitational redshift of spectral lines, Energy-momentum tensor of a perfect fluid, Schwarzschild internal solution, Boundary conditions, Action Principle, Derivation of Einstein field equations from variational principle, Mach's principle

**Unit-III:**

**Marks: 15**

Static cosmological models of Einstein and De-Sitter, their derivation, properties and comparison with the actual universe. Non-static cosmological models, Derivation of Robertson-Walker metric, Hubble's law, Cosmological principles, Weyl's postulate.



**Unit-IV:****Marks: 15**

Friedmann models, Fundamental equations of dynamical cosmology, Critical density, Closed and open universes, Age of the universe, Matter dominated era of the universe, Einstein-de Sitter model, Particle and event horizons, Perfect cosmological principle, Steady state cosmology.

**Reference Books:**

- [1] Introduction to General Relativity - Ronald Ader, Maurice Bazin, Menahem Schiffer, 2 Edition, McGraw Hill Company.
- [2] General Relativity and Cosmology - J. V. Narlikar, Macmillan Company of India, 1978.
- [3] Gravitation and Cosmology: Principles and Applications of General Theory of Relativity - Steven Weinberg, John Wiley Publication.
- [4] Introduction to General Relativity, McGraw Hill Inc., 1975.
- [5] B. Schutz, Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity, John Wiley & Sons, Inc., 1972.
- [6] R. K. Sachs and H. Wu., General Relativity for Mathematician, Springer Verlag, 1977.

**SEMESTER - IV****Paper Code: MATSPL 25074 (C)****Paper Title: Category Theory II****Theory Marks: 70****Internal Marks: 30**

**Course Outcome:** Study about Convergence structure in Category Theory. Axiomatization of short exact sequences in additive categories.

**Unit-I:****Marks: 15**

Exponential in a category, Heyting algebra, ccc category,  $\lambda$ -calculus.

**Unit-II:****Marks: 20**

Set-valued functor categories, The Yoneda embedding, The Yoneda Lemma, Applications of the Yoneda lemma, Limits, Colimits and Exponentials in Categories of diagrams.  $\text{Hom}(X, G^P)$  and  $\text{Hom}(X \times P, Q)$ .

**Unit-III:****Marks: 20**

Adjunction between categories, left and right adjoints, Hom-Set definition of adjoints. Examples of Adjoints, Uniqueness up to isomorphism. Order Adjoints and interior operation in Topology as an order adjoint. Preservation of Limits (Co limits) by Right (Left) Adjoints. UMP of the Yoneda Embedding and Kan Extensions. Statement only of the Adjoint Functor Theorem.

**Unit-IV:****Marks: 15**

Normal and exact categories, additive categories, abelian categories.

**Text Books:**

- [1] Awodey, S.: Category Theory, (Oxford Logic Guides, 49, Oxford University Press.)

**Reference Books:**

- [1] Herrlich, Horst; Strecker, George E. (2007), Category Theory (3rd ed.), Heldermann Verlag Berlin

**SEMESTER - IV****Paper Code: MATSPL 25084 (A)****Paper Title: Number Theory II****Theory Marks: 70****Internal Marks: 30**

**Course Outcome:** Introduction to the arithmetic functions and their properties. Study on congruence theory, distribution of prime numbers and partition theory.

**Unit-I:****Marks: 20**

Arithmetical functions and Dirichlet multiplication, averages of arithmetical functions. Elementary theorems on the distribution of primes, the prime number theorem, Chebyshev's functions and their relations. Dirichlet's theorem for primes.

**Unit-II:****Marks: 15**

Quadratic residues and quadratic reciprocity law, applications of the reciprocity law, Gauss sums.

**Unit-III:****Marks: 20**

Dirichlet series, Euler products, Riemann zeta function and Dirichlet  $L$ -functions.

**Unit-IV:****Marks: 15**

Introduction to partitions, geometric representation, generating functions, Euler's Pentagonal number theorem, Jacobi triple product identity, Jacobi's identity, recursion formula for  $p(n)$ . Ramanujan's partition identities.

**Text Books:**

- [1] Apostol, T. M. Introduction to Analytic Number Theory, Springer International Student Edition (Narosa Publishing House, New Delhi, 1993).  
[2] Hardy, G.H. and Wright, E. M. An Introduction to the Theory of Numbers, 4th Edition (Oxford University Press, 1960).

**Reference Books:**

- [1] Berndt, B.C., Number Theory in the spirit of Ramanujan.  
[2] Niven, I. and Zuckerman, H. An Introduction to the Theory of Numbers, 5th Edition (Wiley Eastern, New Delhi, 2000).  
[3] Andrews, G.E. Number Theory (Hindustan Publishing Corporation, New Delhi, 1992).

## SEMESTER - IV

**Paper Code: MATSPL 25084 (B)**  
**Paper Title: Dynamical Systems II**  
**Theory Marks: 70**  
**Internal Marks: 30**

### **Course Outcome:**

By the end of this course, students will:

- Differentiate between discrete and continuous dynamical systems and analyze iterations and orbits.
- Conduct graphical and orbit analysis, including phase portraits and stability evaluation.
- Identify and analyze fixed points, periodic points, and their stability.
- Study dynamics and bifurcations, including saddle node and period doubling bifurcations.

### **Unit-I:**

**Marks: 20**

Transition of Chaos, Symbolic Dynamics, Chaos, Sharkovskii's Theorem, The role of critical orbits, Newton's Method, Complex Functions, The Julia Set, The Mandelbrot Set.

### **Unit-II:**

**Marks: 20**

**The Poincaré Map:** Introduction to the Poincaré map and its significance in dynamical systems, Construction, and properties of the Poincaré map, Applications in periodic orbits, stability analysis, and chaos theory.

**Gortman-Hartman Theorem:** Statement and proof of the Gortman-Hartman theorem, Implications for stability analysis of dynamical systems, Applications in nonlinear dynamics and control theory.

**Phase Plane:** Classification of linear systems, Lyapunov stability, Structural stability, Examples and applications, Problems.

### **Unit-IV:**

**Marks: 20**

**Nonlinear Oscillators:** Introduction to nonlinear oscillators, The Duffing oscillator: Lindstedt's method, elliptic functions, and problem-solving, The van der Pol oscillator: Method of averaging, Hopf bifurcations, homoclinic bifurcations, relaxation oscillations, and problem-solving.

### **Unit-V:**

**Marks: 10**

**Fractals:** Definition of fractals and fractal geometry, Historical background and key contributors, Self-similarity and fractal dimension, the cantor set revisited, Sierpinski triangle, Koch Snowflake, Topological dimension, Fractal dimension.

### **Reference Books:**

- [1] Differential Equation, Dynamical System and Linear Algebra: Morris W. Hirsch, Stephen Smale, Academic Press.
- [2] An introduction to Dynamical systems: D. K. Arrowsmith
- [3] A First Course in Chaotic Dynamical Systems: Robert L. Devaney
- [4] Chaos and Fractals: New Frontiers of Science: Heinz-Otto Peitgen, Hartmut Jürgens, and Dietmar Saupe.

- [5] Nonlinear Dynamics and Chaos: With Applications to Physics, Biology, Chemistry, and Engineering: Steven H. Strogatz
- [6] Dynamical Systems: Stability, Symbolic Dynamics, and Chaos: Clark Robinson
- [7] An Introduction to Dynamical Systems and Chaos: G.C. Layek
- [8] Nonlinear Dynamics and Chaos: Geometric Methods for Engineers and Scientists by J. M. T. Thompson and H. B. Stewart
- [9] Dynamical Systems and Chaos by Mauricio Matos Peixoto and David A. Rand

## **SEMESTER - IV**

**Paper Code: MATSPL 25094 (A)**

**Paper Title: Analysis III**

**Theory Marks: 70**

**Internal Marks: 30**

**Course Outcome:** Study about Lebesgue measure over general measure spaces, productivity on  $\mathbb{R}^n$ , with the topological properties.

**Unit-I:** **Marks: 15**

General Measure Theory: Measure space, completion of a measure space, signed measure, Hahn and Jordan decomposition theorem for finite signed measure, extension of a premeasure to a measure, Carathéodory-Hahn theorem, Randon-Nikodym theorem for finite and  $\sigma$ -finite signed measures and its consequences, computation of the Randon-Nikodym derivative, Nikodym metric space, Vitali-Hahn-Saks Theorem.

**Unit-II:** **Marks: 15**

General  $L^p$  Space: Completeness, duality and weak convergence.

**Unit-III:** **Marks: 20**

Product measures: Fubini and Tonelli theorem, Lebesgue measure on Euclidean space  $\mathbb{R}^n$  Lebesgue-Stieltjes measures and Lebesgue-Stieltjes integral, Hausdorff measure and its dimension.

**Unit-IV:** **Marks: 20**

Measure and Topology: Construction of Radon measure, Riesz-Markov theorem, Riesz representation theorem for the dual of  $C(X)$ , regularity of Baire measures, Borel measure on compact groups, Von Neumann's theorem, Bogoliubov-Krilov theorem.

### **Text Books:**

- [1] Royden, H. L., Fitzpatrick, P. M., Real Analysis, 4<sup>th</sup> Edition, PHI Learning Private Limited, New Delhi, 2011

### **Reference Books:**

- [1] Munroe, M. E.: Introductions to measure and integration, Addison Wesley, 1953.
- [2] Berberian, S. K.: Measure and integration, Chelsa Pub. Co. N.Y. 1965.
- [3] Ruddin, W.: Real and Complex Analysis, Tata McGraw Hill, New Delhi, 1974

## **SEMESTER - IV**

**Paper Code: MATSPL 25094 (B)**

**Paper Title: Biomathematics**

**Theory Marks: 70**

**Internal Marks: 30**

### **Course Learning Objectives:**

1. Comprehend continuous population models for single species, including exponential and logistic growth models, and their qualitative analysis.
2. Learn to perform stability and equilibrium analysis of various population growth models, including those with delays.
3. Understand and model interactions between species using continuous models, focusing on competition, mutualism, and predator-prey dynamics.
4. Gain insights into the impact of harvesting on natural populations, including models with delayed recruitment and age distribution.
5. Utilize mathematical techniques such as differential equations and qualitative analysis to study and solve population dynamics problems.

### **Course Learning Outcomes:**

1. Demonstrate the ability to model single-species population growth using exponential, Malthusian, and logistic growth models, and perform qualitative analysis of these models.
2. Analyze continuous single-species population models with delays, understand the general delay model, and perform stability and equilibrium analysis.
3. Apply continuous models to study interactions between species, including competition, mutualism, and predator-prey dynamics using Lotka-Volterra models.
4. Evaluate the effects of different harvesting strategies on population dynamics, including constant effort and constant yield harvesting, and analyze models with age distribution.
5. Analyze predator-prey interactions using Lotka-Volterra models, understand the role of density dependence, and study classic laboratory experiments and natural system predation.

### **Unit-I:**

**Marks: 20**

Continuous population Model for single species, Exponential population growth model, continuous population growth model: Malthus model for population growth, General population growth model, Qualitative Analysis: Equilibrium points, Stability Analysis, Logistic population growth model and their qualitative analysis, logistic growth model for non-isolated population.

### **Unit-II:**

**Marks: 20**

Insect Out-break model: Spruce Budworm, Continuous Single species population model with Delays: Introduction, General Delay model and Qualitative Analysis, Logistic model with time delay effects, Definition of stability, equilibrium points & stability Analysis.

### **Unit-III:**

**Marks: 10**

Continuous Models for Interacting Population: Interaction between species: two species models, community matrix approach, Qualitative behaviour of community matrix,

Competition: Lotka-Volterra models, Competition Models, Principle of competitive exclusion, Models for Mutualism.

**Unit-IV:** **Marks: 10**

Harvesting a single Natural population: Harvesting in Delayed recruitment models: Constant effort Harvesting, constant yield harvesting, population model with Age Distribution, Simple Discrete population model.

**Unit-V:** **Marks: 10**

Predator: Prey interaction: Lotka-Volterra models, dynamic of simple Lotka-Volterra models, Role of density dependent in the Prey, Classic laboratory experiment on predator, predation in natural system. Some predator- prey models.

**Reference Books:**

- [1] J. N. Kapur, Mathematical Modelling, New Age International Publishers.
- [2] J.D. Murray Mathematical Biology (An Introduction, Vol. I & II), Springer- Verlag.
- [3] J.N. Kapur, Mathematical Model in Biology and Medicines.
- [4] S. I. Rubinow, Introduction to Mathematical Biology, John Wiley and Sons, 1975.
- [5] MA Khanday, Introduction to Modeling and Biomathematics, Dilpreet Publishers New Delhi, 2016.
- [6] Jaffrey R. Chasnov, Mathematical Biology, Hong Kong Press.

**SEMESTER - IV**

**Paper Code: MATSPL 25104 (A)**

**Paper Title: Network Theory**

**Theory Marks: 70**

**Internal Marks: 30**

**Course Outcome:** This course is about the networks involving graphs. The course deal with various types of complex and random networks. This course also included the basic of Graph neural networks.

**Unit-I:** **Marks: 15**

Overview of graph network, Network Flow: Max-flow Min-cut and the Ford-Fulkerson algorithm.

**Unit-II:** **Marks: 20**

Complex Networks: Introduction to complex networks, Network traversal, Construction of Euler tour, Finding a Hamilton cycle, Trees in transportation networks, Routing in communication networks.

**Unit-III:** **Marks: 10**

Random networks, Computer networks; Social networks analysis; Structural balance; Affiliation networks; Equivalence; Structural equivalence.

**Unit-IV:****Marks: 25**

Representation Learning, Graph representation learning, Introduction to Graph Neural network (GNN), Convolutional neural network (CNN), General Framework of GNN and CNN, Types of GNN: Recurrent Graph Neural Network (RECGNN or RGNN), Graph Attention Network (GAN), Applications of GNN.

**Books:**

- [1] Introduction to Graph Theory, D. B. West, Prentice-Hall, (2001).
- [2] Graph Theory and Complex Networks: An Introduction, Maarten van Steen, (2010).
- [3] Networks, Crowds, and Markets: Reasoning about a Highly Connected World, David Easley and Jon Kleinberg. Cambridge University Press, (2010).
- [4] Introduction to Graph Neural Networks, Zhiyuan Liu and Zhou Jie, Springer Nature, (2022).
- [5] Graph Neural Networks: Foundations, Frontiers, and Applications, Jian Pei (Editor), Liang Zhao (Editor), Lingfei Wu (Editor), Peng Cui (Editor), Springer, (2022).
- [6] Graph Representation Learning, William L. Hamilton, McGill University, (2020).

**SEMESTER - IV****Paper Code: MATSPL 25104 (B)****Paper Title: Fluid Dynamics II****Theory Marks: 70****Internal Marks: 30****Course Learning Objectives:**

1. Gain a comprehensive understanding of wave dynamics, including the behavior of waves at the interface of two liquids, energy transmission in waves, and the significance of group velocity.
2. Develop a thorough understanding of the Navier-Stokes equations for viscous fluid motion, and study various types of steady laminar flows, including flows between parallel plates and through tubes.
3. Learn the principles of dynamical similarity and dimensional analysis, including the Buckingham Pi-theorem, and understand the significance of Reynolds number and magnetic Reynolds number.
4. Explore the theory of very slow motions, including Stokes and Oseen's equations, and apply these to analyze flow past a sphere and lubrication theory.
5. Gain insights into the theory of laminar boundary layers, including the two-dimensional boundary layer equations for flow over a plane wall, and apply Blasius-Topfer solutions to solve boundary layer problems.

**Course Learning Outcomes:**

1. Demonstrate the ability to analyze energy in stationary and progressive waves, calculate group velocity, and understand its dynamical significance in simple harmonic surface waves.
2. Apply the Navier-Stokes equations to solve problems involving steady laminar flow between parallel plates, through circular and non-circular tubes, and understand various flow scenarios such as Plane Couette and Poiseuille flow.

3. Utilize principles of dimensional analysis and the Buckingham Pi-theorem to solve fluid dynamics problems, and explain the significance of Reynolds number and its applications in different fluid flow conditions.
4. Solve problems involving very slow motions using Stokes and Oseen's equations, analyze flow past a sphere, and apply lubrication theory to practical fluid dynamics problems.
5. Apply the theory of laminar boundary layers to two-dimensional flow over a plane wall, use Blasius-Topfer solutions to find solutions to boundary layer equations, and understand the implications for practical fluid dynamics scenarios.

**Unit-I:**

**Marks: 20**

Waves, Waves at the interface of two liquids, and with upper surface free, Energy of Stationary Waves and Progressive Waves, Group Velocity, Rate of Transmission of energy in simple harmonic surface waves, Dynamical significance of group velocity.

**Unit-II:**

**Marks: 20**

Navier-Stokes equations of motion of a viscous fluid, Equation of state for perfect fluid, Steady laminar flow between two parallel plates, Plane Couette flow, Plane Poiseuille flow, The Hagen-Poiseuille flow, Laminar steady flow between two coaxial circular cylinders. Laminar steady flow of incompressible viscous fluid in tubes of cross-section other than circular.

**Unit-III:**

**Marks: 15**

Dynamical similarity; Dimensional Analysis, Buckingham Pi-theorem; Reynold's number; Significance of Reynold's number, Maxwell's electromagnetic field equation; Magnetic Reynold's number.

**Unit-IV:**

**Marks: 15**

Theory of very slow motions, Stokes equation, Oseen's equations, flow past a sphere, Lubrication theory. Theory of laminar boundary layers, Two-dimensional boundary layer equations for flow over a plane wall, Blasius-Topfer solutions.

**Reference Books:**

- [1] Chorlton, Text Book of Fluid Dynamics, CBS Publishers, Delhi, 1985.
- [2] G. K. Batchelor, An Introduction to Fluid Mechanics, Foundation Books, New Delhi, 1994.
- [3] H. Schlichting, Boundary Layer Theory, McGraw Hill Book Company, New York, 1971.
- [4] M. D. Rai Singhania, Fluid Mechanics (With Hydrodynamics) S. Chand and Company Ltd., New Delhi.
- [5] R. K. Rathy, An Introduction to Fluid Dynamics, Oxford and IBH Publishing Company, New Delhi, 1976.