NEP 2020

Syllabus of
One-Year Postgraduate Programme
in Mathematics

Bodoland University
Kokrajhar-783370, Assam, India

STRUCTURE OF THE SYLLABUS
ONE-YEAR POST GRADUATE COURSE IN MATHEMATICS
BODOLAND UNIVERSITY

	Class	Paper	Topics		Credit	Marks
1ST YEAR	**SEMESTER -I**	MATSPL 25014	MATSPL 25014 (A)	Fuzzy Set Theory	4	Internal: 30 Theory: 70
			MATSPL 25014 (B)	Fluid Dynamics I		
			MATSPL 25014 (C)	Advanced Topology I		
		MATSPL 25024	MATSPL 25024 (A)	Graph Theory II	4	Internal: 30 Theory:70
			MATSPL 25024 (B)	Special Theory of Relativity		
			MATSPL 25024 (C)	Category Theory I		
		MATSPL 25034	MATSPL 25034 (A)	Number Theory I	4	Internal: 30 Theory: 70
			MATSPL 25034 (B)	Dynamical Systems I		
		MATSPL 25044	MATSPL 25044 (A)	Advanced Functional Analysis	4	Internal: 30 Theory: 70
			MATSPL 25044 (B)	Numerical Analysis I		
		MATSPL 25054	Computer Lab II		4	Internal: 30 Practical: 70
	SEMESTER -II	MATSPL 25064	MATSPL 25064(A)	Fuzzy Logic and Control System	4	Internal: 30 Theory: 70
			MATSPL 25064(B)	Numerical Analysis II		
			MATSPL 25064(C)	Advanced Topology II		
		MATSPL 25074	MATSPL25074 (A)	Graph Theory III	4	Internal: 30 Theory: 70
			MATSPL25074 (B)	Relativity and Cosmology		
			MATSPL25074 (C)	Category Theory II		
		MATSPL 25084	MATSPL 25084 (A)	Number Theory II	4	Internal: 30 Theory: 70
			MATSPL 25084 (B)	Dynamical Systems II		
		MATSPL 25094	MATSPL 25094 (A)	Analysis III	4	Internal: 30 Theory: 70
			MATSPL 25094 (B)	Biomathematics		
		MATSPL 25104	MATSPL 25104 (A)	Network Theory	4	Internal: 30 Theory: 70
			MATSPL 25104 (B)	Fluid Dynamics II		

Detailed Syllabus

SEMESTER - I

Paper Code: MATSPL 25014 (A)
Paper Title: Fuzzy Set Theory
Theory Marks: 70
Internal Marks: 30

Curse Outcomes: This course will help the students to learn about the fuzzy set theory and recent practical development of fuzzy set theory in the present environment.

Unit-I: **Marks: 30**
Fuzzy sets - basic definitions, α-level sets, convex fuzzy sets, basic operations on fuzzy sets, types of fuzzy sets, Cartesian products, algebraic products, bounded sum and difference, t-norms and t-conforms. Fuzzy sets in contrast of probability theory.

Unit-II: **Marks: 20**
The extension principle - the Zadeh's extension principle, image and inverse image of fuzzy Sets, fuzzy numbers, elements of fuzzy arithmetic.

Unit-III: **Marks: 20**
Fuzzy relations and fuzzy graphs, composition of fuzzy relations, min-max composition and its properties, fuzzy equivalence relations, fuzzy relation equations, fuzzy graphs.

Reference Books:

[1] Zimmermann, H. J., Fuzzy set theory and its Applications, Allied publishers Ltd., New Delhi, 1991.
[2] Klir, G. J. and Yuan, B., Fuzzy Sets and Fuzzy Logic: Theory and Applications, Prentice Hall of India, New Delhi, 1997.

SEMESTER - I

Paper Code: MATSPL 25014 (B)
Paper Title: Fluid Dynamics I
Theory Marks: 70
Internal Marks: 30

Course Learning Objectives:
1. Comprehend the fundamental properties and dynamics of real and ideal fluids, including velocity, acceleration, and continuity equations.
2. Learn to derive and apply Bernoulli’s theorem in various fluid flow scenarios, understanding its implications for fluid motion and energy conservation.
3. Develop skills to analyze two-dimensional fluid motion using complex velocity potentials, conformal mapping, and specific flow patterns around objects.
4. Gain insights into the stress components in real fluids, the relationship between stress and velocity gradients, and perform stress analysis in fluid motion.

5. Study vortex motion, including circular and rectilinear vortices, and understand the concepts of boundary layer theory, including boundary layer thickness and associated properties.

Course Learning Outcomes:

1. Demonstrate the ability to model and analyze fluid flow, using concepts such as velocity potential, streamlines, path lines, and streak lines, and apply Lagrange's and Euler's equations of motion.
2. Solve problems involving two-dimensional fluid motion using complex velocity potentials, sources, sinks, doublets, and conformal mapping techniques.
3. Conduct stress analysis in fluid motion, relating stress components to velocity gradients and understanding the rate of strain quadric of fluid elements.
4. Analyze elementary properties of vortex motion, apply Kirchhoff's vortex theorem, and solve problems involving motions due to circular and rectilinear vortices.
5. Apply boundary layer theory concepts to practical problems, calculate boundary layer thickness, displacement, momentum, and energy thickness, and understand their implications for fluid flow near boundaries.

Unit-I: **Marks: 20**

Real fluids and Ideal fluids; Velocity and Acceleration of a fluid particle; Equation of continuity; Boundary Surface; Stream lines, Path lines and streak lines; Velocity potential; Equation of motion: Langrange's and Euler's equations of motion; Bernoulli's Theorem.

Unit-II: **Marks: 20**

Motion in two-dimensions; Complex velocity potential; Sources, sinks, doublets and their images; Conformal mapping; Milne-Thomson circle theorem; Two-dimensional irrotational motion produced by motion of circular, co-axial and elliptic cylinders in an infinite mass of liquid; Theorem of Blasius; Stoke's stream function.

Unit-III: **Marks: 15**

Stress components in a real fluid; Relations between rectangular components of stress. Connection between stresses and gradients of velocity, Stress analysis in fluid motion, Rate of strain quadric of fluid element, Transformation of rate of strain components.

Unit-IV: **Marks: 15**

Vortex motion and its elementary properties; Motions due to circular and rectilinear vertices; Kirchhoff vortex theorem. Boundary layer theory: Boundary layer concept, boundary layer thickness, displacement, momentum, and energy thickness.

Reference Books:

[1] Fluid Dynamics - William F. Hughes, John A. Brighton, Schaum's Outline Series, McGraw-Hill.
[2] Boundary-Layer Theory - Hermann Schlichting, Klaus Gersten, Springer.
[3] Chorlton, Text Book of Fluid Dynamics, CBS Publishers, Delhi,1985.
[4] G. K. Batchelor, An Introduction to Fluid Mechanics, Foundation Books, New Delhi, 1994.
[5] H. Schlichting, Boundary Layer Theory, McGraw Hill Book Company, New York, 1971.
[6] M. D. Raisinghania, Fluid Mechanics (With Hydrodynamics) S. Chand and Company Ltd., New Delhi.

[7] R. K. Rathy, An Introduction to Fluid Dynamics, Oxford and IBH Publishing Company, New Delhi, 1976.

SEMESTER - I

Paper Code: MATSPL 25014 (C)
Paper Title: Advanced Topology I
Theory Marks: 70
Internal Marks: 30

Course Outcome: Study about the properties of Metric spaces related to convergence structure in topological spaces. Metrization based on paracompactness. The role of fundamental group in the study of topological spaces.

Unit-I: **Marks: 20**
Nets and filters, convergenge in terms of nets and filters, ultrafilters and compactness. Theories of metrization, Urysohn's Lemma, Tietze Extension theorem, Urysohn metrization Theorem.

Unit-II: **Marks: 15**
Paracompactness, characterisation in regular spaces, metrization based on paracompactness, Nagata-Smirnov theorem, Stone’s theorem, Smirnov’s metrization theorem.

Unit-III: **Marks: 20**
Topological groups, subgroups, quotient groups, homogeneous spaces, product groups.

Unit-IV: **Marks: 15**
Homotopy and the fundamental group, computation of the fundamental group of the circle.

Text Books:
[1] Willard, S. General Topology, Addision-Wesley, Reading, 1970
[2] Munkres, J. R.**,** Topology: A first course (2/e), Prentice-Hall, 2000

Reference Books:
[1] Joshi, K. D., Topology, Wiley-Eastern, 1988.
[2] Kelley, J. L., Topology, Van Nostrand, 1955.

SEMESTER - I

Paper Code: MATSPL 25024 (A)
Paper Title: Graph Theory II
Theory Marks: 70
Internal Marks: 30

Course Outcome: This course is about to the topics of graph theory like Factoriztion, planarity, colourability matching and domination. The course introduces basic graph algorithms with applications.

Unit-I: **Marks:15**

Factorization: factors, 1-factorization, 2-factorization, Tutte's 1-factor theorem. Planarity: Graph embeddings; Planar graph, Euler's formula, Parameters of planarity.

Unit-II: **Marks: 20**

Colourability: Vertex and edge colouring, Chromatic number and edge chromatic number, Bounds for chromatic number, Brooks' theorem; Vizing's theorem, The four colour theorem, The five colour conjecture, Uniquely colourable graph.

Unit-III: **Marks: 25**

Matching: Basic concepts of matching, maximum, maximal and perfect matching, Augmented path, Berge's theorem, Matching in bipartite graph, König's theorem for maximum matching. Domination: Dominating set and dominating number, Closed and open neighbourhood, Total dominating set and total domination number.

Unit-IV: **Marks:10**

Basic Graph Algorithms: Traversal algorithm like breadth first search, depth first search, Designing and analyzing of algorithms, Topological sort.

Books:

[1] Introduction to Graph Theory, D. B. West, Prentice-Hall, (2001).
[2] Basic Graph Theory, K. R. Parthasarathy, Tata McGraw Hill, (1994)
[3] Graph Theory, F. Harary, Narosha Publishing Co (Reprint 1998).
[4] Algorithm Design: J. Kleinberg and E. Tardos (2006) Pearson Education.
[5] Introduction to Algorithms: H. Cormen, C.E. Leiserson and R.L. Rivest (2009) MIT Press.

SEMESTER - I

Paper Code: MATSPL 25024 (B)
Paper Title: Special Theory of Relativity
Theory Marks: 70
Internal Marks: 30

Course Learning Objectives:

1. Develop an understanding of inertial and non-inertial frames, the geometry of Newtonian mechanics, and the transformations between these frames, particularly Galilean transformations.
2. Comprehend the fundamental postulates of the special theory of relativity, including Lorentz transformations and their implications for physical phenomena such as length contraction, time dilation, and simultaneity.
3. Study the variation of mass with velocity, the equivalence of mass and energy, and the transformations of mass, force, acceleration, density, momentum, and energy in relativistic contexts, including the formulation of relativistic Lagrangian and Hamiltonian mechanics.
4. Gain insights into Minkowski space and its role in representing relativistic concepts geometrically, including four-vectors, four-velocity, four-force, and four-momentum, and the covariant four-dimensional formulation of the laws of mechanics.
5. Learn the fundamentals of electrodynamics within the framework of special relativity, including the transformation of differential operators, Maxwell's equations,

electromagnetic waves, and the transformation properties of electric and magnetic fields.

Course Learning Outcomes:

1. Demonstrate the ability to apply Galilean and Lorentz transformations to solve problems in Newtonian mechanics and special relativity, including calculating length contraction, time dilation, and relativistic velocity addition.
2. Solve problems involving the variation of mass with velocity, the equivalence of mass and energy, and transform physical quantities such as mass, force, acceleration, density, momentum, and energy using relativistic principles.
3. Use Minkowski space to represent and analyze relativistic phenomena, including constructing and interpreting four-vectors, and solving relativistic equations of motion using covariant four-dimensional formulations.
4. Apply the principles of electrodynamics within the context of special relativity to transform differential operators, solve Maxwell's equations, analyze electromagnetic waves, and understand the transformations of electric and magnetic field components.
5. Integrate concepts from special relativity and electrodynamics to solve complex problems, demonstrating a thorough understanding of the interplay between relativistic mechanics and electromagnetic theory.

Unit-I: **Marks: 20**

Inertial and non-inertial frames, Geometry of Newtonian mechanics, Galilean Transformations, Fundamental postulates of the special theory of relativity, Lorentz transformation, Lorentz transformation as a group. Length contraction, Time dilation and Simultaneity. Relativistic addition law of velocities.

Unit-II: **Marks: 15**

Relativistic mechanics, Variation of mass with velocity, Equivalence of mass and energy. Transformation of mass, force acceleration, density, momentum and energy. Relativistic Lagrangian and Hamiltonian.

Unit-III: **Marks: 15**

Minkowski's space, Geometrical representation of simultaneity, contraction and time dilation, Space-like and time- like intervals, Position four vectors, four-velocity, four forces and four momentums, Relativistic equations of motion. Covariant four-dimensional formulations of laws of mechanics.

Unit-IV: **Marks: 20**

Electrodynamics: Fundamental of electrodynamics, Transformation of differential operators, D'Alembert operator, Maxwell's equations and electromagnetic waves, Electromagnetic potentials and force Lorentz condition, Transformations of charge and current density, transformation equations of electric field strength and magnetic field induction components.

Reference Books:

[1] Introduction to Special Relativity, Wiley Eastern Ltd. (1990) Robert.
[2] The Mathematical Theory of Relativity, Cambridge University Press 1965 A. S. Eddington.
[3] Relativity, Thermodynamics and Cosmology - R.C. Tolman (Oxford Press)

SEMESTER - I

Paper Code: MATSPL 25024 (C)
Paper Title: Category Theory I
Theory Marks: 70
Internal Marks: 30

Course Outcome: Study about the fundamental structure and basic properties of Category Theory as Categories are algebraic structures with many complementary natures, e.g., geometric, logical, computational, combinatorial, just as groups are many-faceted algebraic structures.

Unit-I: **Marks:15**
Definition and examples ofcategories, The concept of functor and the category Cat. Natural Transformations, isomorphism, epimorphism, monomorphism, Mobics, Epis and Zeros Foundations, Large Categories, Hom-sets. Free Categories, Constructions of new categories from old categories, The Dual Category, The Arrow Category, The Slice and Co- Slice Category.

Unit-II: **Marks: 20**
Epis and mono, Initial and Terminal objects, Generalized elements, Sections and Retractions, Categories of categories, properties of functors, natural transformation and natural isomorphism, isomorphisms and equivalences of categories, functor categories.

Unit-III: **Marks: 20**
Groups in categories. The category of groups, Groups as categories, Congruence on a category, quotient category and its univalent mapping property, finitely presented categories.

Unit-IV: **Marks:15**
Equalizers and coequalizers, intersections and factorizations, products and coproducts, sources and sinks, limits and colimits, pullback and pushout, inverse and direct limits, complete categories, limits in factor categories.

Text Books:

[1] Awodey, S.: Category Theory, (Oxford Logic Guides, 49, Oxford University Press.)

Reference Books:

[1] Herrlich, Horst; Strecker, George E. (2007), Category Theory (3rd ed.), HeldermannVerlag Berlin

SEMESTER - I

Paper Code: MATSPL 25034 (A)
Paper Title: Number Theory I
Theory Marks: 70
Internal Marks: 30

Course Outcome: Basic study of arithmetic number theory including divisibility, congruences, linear Diophantine equation.

Unit-I: **Marks: 15**

Divisibility, greatest common divisor, least common multiple, Euclidean Algorithm. Chinese Remainder Theorem. Prime numbers, factorization in prime numbers, fundamental theorem of arithmetic.

Unit-II: **Marks: 15**

Divisor functions, perfect numbers, Mersenne numbers, Fermat numbers.

Unit-III: **Marks: 20**

Greatest integer function (Gauss function), Mobius function, Euler function. Congruences and its elementary properties, congruences in one unknown, complete residue system, reduced residue system.

Unit-IV: **Marks: 20**

Diophantine equations, linear Diophantine equations, Pythagoras equation, sum of two squares. Quadratic residues and congruences of second degree in one unknown, Legendre symbol, Jacobi symbol, congruences of second degree with prime modulus and with composite modulus.

Text Books:

[1] Hardy, G.H. and Wright, E. M. An Introduction to the Theory of Numbers, 4th Edition (Oxford University Press, 1960).
[2] Andrews, G.E. Number Theory (Hindustan Publishing Corporation, New Delhi, 1992).

Reference Books:

[1] Burton, D. M. Elementary Number Theory, 6th Edition (Tata McGraw-Hill, NewDelhi,2007).
[2] Niven, I. and Zuckerman, H. An Introduction to the Theory of Numbers, 5th Edition (WileyEastern, New Delhi, 2000).

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SEMESTER - I

Paper Code: MATSPL 25034 (B)
Paper Title: Dynamical Systems I
Theory Marks: 70
Internal Marks: 30

Unit-I: **Marks: 20**

Dynamical systems, discrete and continuous dynamical systems, Examples of dynamical systems. Iteration, Orbits, Types of Orbits, Other Orbits, The Doubling Function.

Unit-II: **Marks: 25**

Graphical Analysis, Orbit Analysis, The Phase Portrait, Examples, Solutions Fixed and Periodic Points, A Fixed-Point Theorem, Attraction and Repulsion. Stability of fixed points and equilibrium points.

Unit-III: **Marks: 25**

Bifurcations: Dynamics of the Quadratic Maps, Saddle Node Bifurcation, Period Doubling Bifurcations, The Quadratic Family

Reference Books:

[1] Differential Equation, Dynamical System and Linear Algebra: Morris W. Hirsch, StephenSmale, Academic Press.
[2] An introduction to Dynamical systems: D. K. Arrowsmith
[3] A First Course in Chaotic Dynamical Systems: Robert L. Devaney

<u>SEMESTER - I</u>

Paper Code: MATSPL 25044 (A)
Paper Title: Advanced Functional Analysis
Theory Marks: 70
Internal Marks: 30

Course Outcome: This course is about the basic of Topological vector spaces, Banach and C* algebras. The course deals with various properties of different types of linear operators. This course also included fixed point theory

Unit-I: **Marks: 20**

Topological Vector Spaces: Introduction, Separation properties, Linear mappings, Finite-dimensional spaces, Metrization, Boundedness and continuity, Seminorms and local convexity, Quotient space, completeness, convexity, Weak topologies, Compact convex sets.

Unit-II: **Marks: 20**

Preliminaries on Banach Algebras and C* Algebras, Commutative Banach Algebras and Commutative C* Algebras, Representation of C* Algebras.

Unit-III: **Marks:20**

Spectral Theory of linear operators in normed space, Spectral Properties of Bounded Linear Operators, Further Properties of Resolvent, Spectrum and Banach Algebras. Spectral Properties of Bounded Self-Adjoint Linear Operators, Positive Operators, Square Roots of a Positive Operator, Projection Operators, Spectral Family of a Bounded Self-Adjoint Linear, Spectral Representation of Bounded Self-Adjoint Linear Operators.

Unit-IV: **Marks: 10**

Fixed Point Theorems and Some Applications to Analysis.

Books:

[1] Kreyszig E., Introductory Functional Analysis with Applications (John Wiley and Sons, New York, 1978).
[2] Rudin, W. Functional Analysis (McGraw Hill, 2000).
[3] Simmons, G. F. Introduction to Topology and Modern Analysis (Tata McGraw Hill Book Co. Ltd.,1963).
[4] Gerard J. Murphy, C* - Algebras and Operator Theory, Academic Press, Inc, 1990.

[5] Ronald G. Douglas, Banach Algebra Techniques in Operator Theory, Second Edition, Springer-Verlag, New York, Inc, 1998.
[6] Limaye, B. V. Functional Analysis (Wiley Eastern Ltd., New Delhi, 1989).

SEMESTER - I

Paper Code: MATSPL 25044 (B)
Paper Title: Numerical Analysis I
Theory Marks: 70
Internal Marks: 30

Course Outcome:
By the end of this course, students will:

- Solve linear equations using LU and Cholesky decompositions, and iterative methods.
- Analyze eigenvalues and eigenvectors with methods like the power method and QR algorithm.
- Solve non-linear equations using Newton-Raphson and Steffensen's iteration.
- Perform numerical integration with Gauss-Legendre, Euler-Maclaurin, and Romberg techniques.

Unit-I: **Marks: 20**
Direct method for solving of linear equations (Crout's method, LU decomposition, Cholesky decomposition), Iterative methods (Relaxation method)., Escalator method Steepest descent and conjugate gradient method.

Unit-II: **Marks: 20**
Algebraic Eigen value problem: Properties of Eigen values and Eigen vectors, Power method, Inverse power method, Jacobi's method, Given's method. Orthogonal factorization, QR algorithm for Eigen value problem Eigen values of complex matrix and complex Eigen vectors.

Unit-III: **Marks: 15**
System of non-linear method equations: Solution of Non-linear Equations: Single Equation: Modified Newton-Raphson method (for real roots-simple or repeated). Aitken's Δ^2-method and Steffensen's iteration. Bairstow's method of quadratic factors, Graeffe's root squaring method. Non-Linear Systems of Equations: Newton's method, Quasi-Newton's method.

Unit-IV: **Marks: 15**
Numerical Integration: Gauss- Legendre and Gauss-Chebyshev Quadrature's, Euler-Maclaurin summation formula, Romberg integration.

Reference Books:
[1] An introduction to Numerical Analysis: Kendal E. Atkinson, Johan Wiley and sons, Inc.
[2] Numerical Methods in engineering & Science: Dr. B. S. Grewal
[3] Introduction to Numerical analysis: C. E. Froberg, Addison Wesley publishing Company, sixth edition, 1981.
[4] Introductory Methods of Numerical Analysis: S. S. Sastri, Prentice Hall of India, New Delhi, 1997.

SEMESTER - I

Paper Code: MATSPL 25054
Paper Title: Computer Lab II
Practical Marks: 70
Internal Marks: 30

Unit-I: Introduction to Numerical Computation

- Review of Numerical Computation Fundamentals
- Advanced Error Analysis
- Introduction to High-Performance Computing
- Parallel and Distributed Computing
- GPU Computing

Unit-II: Solving Nonlinear Equations and Optimization

- Multivariate Root Finding Methods
- Nonlinear Optimization Techniques
- Constrained Optimization
- Global Optimization Methods

Unit-III: Numerical Linear Algebra II

- Singular Value Decomposition (SVD)
- Iterative Methods for Linear Systems
- Sparse Matrix Computations
- Applications in Machine Learning and Big Data Analytics

Unit-IV: Advanced Topics in Computational Mathematics

- Numerical Methods for Partial Differential Equations (PDEs)
- Spectral Methods
- Finite Element Methods for PDEs
- Applications in Computational Fluid Dynamics, Solid Mechanics, and Electromagnetics.

Textbook:

[1] P. Jones, Python: The Fundamentals of Python Programming, CreateSpace Independent Pub.
[2] S. Linge, H. P. Langtangen, Programming for computations, Springer.
[3] E. V. Krishnamurthy and S. K. Sen: Computer Based Numerical Algorithms, East-West press Pvt. Ltd. 1976.
[4] R. Johansson, Numerical Python, A Press.
[5] Numerical Methods for Engineers by Steven C. Chapra and Raymond P. Canale

References:

[1] Numerical Recipes: The Art of Scientific Computing by William H. Press et al.
[2] Introduction to Scientific Computing: A Matrix-Vector Approach Using MATLAB by Charles F. Van Loan.

SEMESTER - II

Paper Code: MATSPL 25064 (A)
Paper Title: Fuzzy Logic and Control System
Theory Marks: 70
Internal Marks: 30

Course outcomes: This course will be beneficial for the students to learn about the application of fuzzy logic and its application. Also, this course will give the idea about multi criteria decision making.

Unit-I: **Marks: 15**

Fuzzy logic, fuzzy propositions, fuzzy quantifiers, linguistic variables, inference from conditional fuzzy propositions, compositional rule of inference.

Unit-II: **Marks: 20**

Approximate reasoning - an overview of fuzzy expert systems, fuzzy implications and their selection, multi-conditional approximate reasoning, role of fuzzy relation equation.

Unit-III: **Marks: 15**

An introduction to fuzzy control - fuzzy controllers, fuzzy rule base, fuzzy inference engine, fuzzification, defuzzification and the various defuzzification methods.

Unit-IV: **Marks: 20**

Decision making in fuzzy environment - individual decision making, multi-person decision making, multi-criteria decision making, multistage decision making, fuzzy ranking methods, fuzzy linear programming, fuzzy logic as a tool in soft computing.

Reference Books:

[1] Zimmermann, H. J., Fuzzy set theory and its Applications, Allied publishers Ltd., New Delhi, 1991.
[2] Klir, G. J. and Yuan, B., Fuzzy Sets and Fuzzy Logic: Theory and Applications, Prentice Hall of India, New Delhi, 1997.

SEMESTER - II

Paper Code: MATSPL 25064 (B)
Paper Title: Numerical Analysis II
Theory Marks: 70
Internal Marks: 30

Course Outcome:

By the end of this course, students will:

- Solve ordinary differential equations using methods like Modified Euler’s and Adams-Bashforth.
- Address boundary-value problems with numerical techniques and the finite-difference method.

- Develop finite-difference approximations and solve elliptic equations for partial differential equations.
- Use relaxation methods and appropriate techniques for parabolic and hyperbolic equations.
- Apply different types of approximations, including least square polynomial and Chebyshev polynomials.

Unit-I: **Marks: 15**

Numerical Solution of Ordinary Differential equations: Modified Euler's method, Predictor-corrector method, Milne's method, Adams-Bash forth method, Boundary-value problems, Finite-difference method.

Unit-II: **Marks: 20**

Numerical Solution of Partial Differential equations: Finite-Difference approximations to partial derivatives, Elliptic equations, Solution of Laplace equation, Solution of Poisson's equation.

Unit-III: **Marks: 20**

Solution of Elliptic Equations by Relaxation Method, Parabolic Equations, Hyperbolic Equations.

Unit-IV: **Marks: 15**

Approximations: Different types of approximation, least square polynomial approximation, polynomial approximation by use of orthogonal polynomials, approximation with Chebyshev polynomials.

Reference Books:

[1] Numerical Methods in Engineering & Science: Dr. B. S. Grewal
[2] An introduction to Numerical Analysis: Kendal E. Atkinson, Johan Wiley and sons, Inc.
[3] Introduction to Numerical analysis: C. E. Froberg, Addision Wesley publishing Company, sixth edition, 1981.
[4] Introductory Methods of Numerical Analysis: S. S. Sastri, Prentice Hall of India, New Delhi, 1997.

SEMESTER - II

Paper Code: MATSPL 25064 (C)
Paper Title: Advanced Topology II
Theory Marks: 70
Internal Marks: 30

Course Outcome: Study about the notion related to the uniform structure in topological spaces. Introduction to the notion of Function space and Proximity structure.

Unit-I: **Marks: 20**

Uniformities, uniform continuity, product uniformities, metrisation, completeness and Compactness, completion.

Unit-II: **Marks: 20**

Uniform structures in Topological groups, complete groups, completion of topological groups.

Unit-III: **Marks: 15**

Function spaces, pointwise convergence, uniform convergence, compact-open topology, k-spaces, equicontinuity, Ascoli theorem.

Unit-IV: **Marks: 15**

Proximity Structure: Smirnov Compactification.

Text Books:

[1] Willard, S. General Topology, Addision-Wesley, Reading, 1970

[2] Munkres, J. R.**,** Topology: A first course (2/e), Prentice-Hall, 2000

Reference Books:

[1] Joshi, K. D. Topology, Wiley-Eastern, 1988.

[2] Kelley, J. L. Topology, Van-Nostrand, 1955.

[3] Bourbaki, N. Elements of Mathematics: General Topology, Vols. I & II,Springer-Verlag, 1988.

SEMESTER - II

Paper Code: MATSPL 25074 (A)
Paper Title: Graph Theory III
Theory Marks: 70
Internal Marks: 30

Course Outcome: This course is an introduction to the generalizations of graphs. The course introduces in an elementary way some basic knowledge of hypergraph and semigraph. This course also included some important algorithms.

Unit-I: **Marks: 20**

Hypergraph: Introduction to Hypergraph, Degree and rank, Incidence matrix, Cycles in hypergraph, conformal hypergraphs, Representative graph of a hypergraph, Dual of hypergraph.

Unit-II: **Marks: 25**

Semigraph: Introduction to Semigraph, Degrees in semigraph; Subsemigraph and partial subsemigraph, Path and cycle, Complete Semigraph, Strongly Complete Semigraph, Edge bipartite, Edge regular, Edge Complete Semigraph, Dendroids (Semitree).

Unit-III: **Marks: 25**

Graph Algorithms: Interval scheduling, Minimum spanning tree, Kruskal’s algorithm, Prim’s algorithm, Single-source Shortest path, The Bellman-Ford algorithm, Dijkstra’s algorithm, Strongly connected components.

Books:

[1] Introduction to Graph Theory, D. B. West, Prentice-Hall, (2001).

[2] Graphs and Hypergraphs, C. Berge, North-Holland, London (1973).
[3] Hypergraphs, C. Berge, North-Holland, London (1973).
[4] Semigraph and Their Application, E. Sampathkumarachar, Academy of DiscreteMathematics and Applications, Lecture Notes, Series No. 1 (2019).
[5] Algorithm Design: J. Kleinberg and E. Tardos (2006) Pearson Education.
[6] Introduction to Algorithms: H. Cormen, C.E. Leiserson and R.L. Rivest (2009) MIT Press

SEMESTER - II

Paper Code: MATSPL 25074 (B)
Paper Title: Relativity and Cosmology
Theory Marks: 70
Internal Marks: 30

Course Learning Objectives:

1. Develop a comprehensive understanding of the foundational principles of general relativity, including the principles of equivalence, covariance, and geodesic principle.
2. Learn to derive and understand Einstein's field equations, including the Newtonian approximation, and analyze solutions such as the Schwarzschild external solution and its isotropic form.
3. Study and interpret various relativistic phenomena, such as redshift, the advance of perihelion of a planet, bending of light rays in a gravitational field, and the energy-momentum tensor of a perfect fluid.
4. Gain insights into static and non-static cosmological models, including the derivation and properties of the Einstein and De-Sitter models, and understand Hubble's law, cosmological principles, and the Robertson-Walker metric.
5. Understand the fundamental equations of dynamical cosmology, explore Friedmann models, and analyze concepts such as critical density, closed and open universes, the age of the universe, and steady state cosmology.

Course Learning Outcomes:

1. Demonstrate the ability to apply the principles of equivalence, covariance, and geodesic principle to solve problems in general relativity and understand their implications for the motion of objects in a gravitational field.
2. Successfully derive Einstein's field equations using variational principles and solve them for specific cases like the Schwarzschild external solution, understanding their physical significance in various relativistic scenarios.
3. Analyze and interpret the effects of general relativity on planetary orbits, the advance of perihelion, bending of light, and gravitational redshift, and understand their observational consequences.
4. Construct static and non-static cosmological models, derive and compare the properties of the Einstein and De-Sitter models with observations of the actual universe, and understand the implications of Hubble's law and the Robertson-Walker metric.
5. Apply the fundamental equations of dynamical cosmology to solve problems related to Friedmann models, calculate critical density, and determine the age and structure of the universe, understanding concepts like particle and event horizons and the steady state cosmological principle.

Unit-I: **Marks: 20**

General Relativity, Principles of equivalence, Principle of covariance, Geodesic principle, Newtonian approximation of equation of motion, Einstein's field equations, Schwarzschild external solution and its isotropic form, Redshift, Tests of general theory of relativity, Planetary orbits and analogues of Kepler's Laws in general relativity.

Unit-II: **Marks: 20**

Advance of perihelion of a planet, Bending of light rays in a gravitational field, Gravitational redshift of spectral lines, Energy-momentum tensor of a perfect fluid, Schwarzschild internal solution, Boundary conditions, Action Principle, Derivation of Einstein field equations from variational principle, Mach's principle

Unit-III: **Marks: 15**

Static cosmological models of Einstein and De-Sitter, their derivation, properties and comparison with the actual universe. Non-static cosmological models, Derivation of Robertson-Walker metric, Hubble's law, Cosmological principles, Weyl's postulate.

Unit-IV: **Marks: 15**

Friedmann models, Fundamental equations of dynamical cosmology, Critical density, Closed and open universes, Age of the universe, Matter dominated era of the universe, Einstein-de Sitter model, Particle and event horizons, Perfect cosmological principle, Steady state cosmology.

Reference Books:

[1] Introduction to General Relativity - Ronald Ader, Maurice Bazin, Menahem Schiffer, 2 Edition, McGraw Hill Company.
[2] General Relativity and Cosmology - J. V. Narlikar, Macmillan Company of India, 1978.
[3] Gravitation and Cosmology: Principles and Applications of General Theory of Relativity - Steven Weinberg, John Wiley Publication.
[4] Introduction to General Relativity, McGraw Hill Inc., 1975.
[5] B. Schutz, Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity, John Wiley & Sons, Inc., 1972.
[6] R. K. Sachs and H. Wu., General Relativity for Mathematician, Springer Verlag, 1977.

SEMESTER - II

Paper Code: MATSPL 25074 (C)
Paper Title: Category Theory II
Theory Marks: 70
Internal Marks: 30

Course Outcome: Study about Convergence structure in Category Theory. Axiomatization of short exact sequences in additive categories.

Unit-I: **Marks: 15**

Exponential in a category, Heyting algebra, ccc category, λ –calculus.

Unit-II: **Marks: 20**

Set-valued functor categories, The Yoneda embedding, The Yoneda Lemma, Applications of the Yoneda lemma, Limits, Colimits and Exponentials in Categories of diagrams. Hom (X, G^P) and Hom (X x P, Q).

Unit-III: **Marks: 20**

Adjunction between categories, left and right adjoints, Hom-Set definition of adjoints. Examples of Adjoints, Uniqueness up to isomorphism. Order Adjoints and interior operation in Topology as an order adjoint. Preservation of Limits (Co limits) by Right (Left) Adjoints. UMP of the Yoneda Embedding and Kan Extensions. Statement only of the Adjoint Functor Theorem.

Unit-IV: **Marks: 15**

Normal and exact categories, additive categories, abelian categories.

Text Books:

[1] Awodey, S.: Category Theory, (Oxford Logic Guides, 49, Oxford University Press.)

Reference Books:

[1] Herrlich, Horst; Strecker, George E. (2007), Category Theory (3rd ed.), Heldermann Verlag Berlin

SEMESTER - II

Paper Code: MATSPL 25084 (A)
Paper Title: Number Theory II
Theory Marks: 70
Internal Marks: 30

Course Outcome: Introduction to the arithmetic functions and their properties. Study on congruence theory, distribution of prime numbers and partition theory.

Unit-I: **Marks: 20**

Arithmetical functions and Dirichlet multiplication, averages of arithmetical functions. Elementary theorems on the distribution of primes, the prime number theorem, Chebyshev's functions and their relations. Dirichlet's theorem for primes.

Unit-II: **Marks: 15**

Quadratic residues and quadratic reciprocity law, applications of the reciprocity law, Gauss sums.

Unit-III: **Marks: 20**

Dirichlet series, Euler products, Riemann zeta function and Dirichlet *L*-functions.

Unit-IV: **Marks: 15**

Introduction to partitions, geometric representation, generating functions, Euler's Pentagonal number. theorem, Jacobi triple product identity, Jacobi's identity, recursion formula for *p(n)*. Ramanujan's partition identities.

Text Books:

[1] Apostol, T. M. Introduction to Analytic Number Theory, Springer International Student Edition (Narosa Publishing House, New Delhi, 1993).
[2] Hardy, G.H. and Wright, E. M. An Introduction to the Theory of Numbers, 4th Edition (Oxford University Press, 1960).

Reference Books:

[1] Berndt, B.C., Number Theory in the spirit of Ramanujan.
[2] Niven, I. and Zuckerman, H. An Introduction to the Theory of Numbers, 5th Edition (WileyEastern, New Delhi, 2000).
[3] Andrews, G.E. Number Theory (Hindustan Publishing Corporation, New Delhi, 1992).

SEMESTER - II

Paper Code: MATSPL 25084 (B)
Paper Title: Dynamical Systems II
Theory Marks: 70
Internal Marks: 30

Course Outcome:

By the end of this course, students will:

- Differentiate between discrete and continuous dynamical systems and analyze iterations and orbits.
- Conduct graphical and orbit analysis, including phase portraits and stability evaluation.
- Identify and analyze fixed points, periodic points, and their stability.
- Study dynamics and bifurcations, including saddle node and period doubling bifurcations.

Unit-I: **Marks: 20**

Transition of Chaos, Symbolic Dynamics, Chaos, Sharkovskii's Theorem, The role of critical orbits, Newtons Method, Complex Functions, The Julia Set, The Mandelbrot Set.

Unit-II: **Marks: 20**

The Poincaré Map: Introduction to the Poincaré map and its significance in dynamical systems, Construction, and properties of the Poincaré map, Applications in periodic orbits, stability analysis, and chaos theory.

Gortman-Hartman Theorem: Statement and proof of the Gortman-Hartman theorem, Implications for stability analysis of dynamical systems, Applications in nonlinear dynamics and control theory.

Phase Plane: Classification of linear systems, Lyapunov stability, Structural stability, Examples and applications, Problems.

Unit-IV: **Marks: 20**

Nonlinear Oscillators: Introduction to nonlinear oscillators, The Duffing oscillator: Lindstedt's method, elliptic functions, and problem-solving, The van der Pol oscillator: Method of averaging, Hopf bifurcations, homoclinic bifurcations, relaxation oscillations, and problem-solving.

Unit-V: **Marks: 10**

Fractals: Definition of fractals and fractal geometry, Historical background and key contributors, Self-similarity and fractal dimension, the cantor set revisited, Sierpinski triangle, Koch Snowflake, Topological dimension, Fractal dimension.

Reference Books:

[1] Differential Equation, Dynamical System and Linear Algebra: Morris W. Hirsch, Stephen Smale, Academic Press.
[2] An introduction to Dynamical systems: D. K. Arrowsmith
[3] A First Course in Chaotic Dynamical Systems: Robert L. Devaney
[4] Chaos and Fractals: New Frontiers of Science: Heinz-Otto Peitgen, Hartmut Jürgens, and Dietmar Saupe.
[5] Nonlinear Dynamics and Chaos: With Applications to Physics, Biology, Chemistry, and Engineering: Steven H. Strogatz
[6] Dynamical Systems: Stability, Symbolic Dynamics, and Chaos: Clark Robinson
[7] An Introduction to Dynamical Systems and Chaos: G.C. Layek
[8] Nonlinear Dynamics and Chaos: Geometric Methods for Engineers and Scientists by J. M. T. Thompson and H. B. Stewart
[9] Dynamical Systems and Chaos by Mauricio Matos Peixoto and David A. Rand

SEMESTER - II

Paper Code: MATSPL 25094 (A)
Paper Title: Analysis III
Theory Marks: 70
Internal Marks: 30

Course Outcome: Study about Lebesgue measure over general measure spaces, productivity on $\mathbb{R}^n$, with the topological properties.

Unit-I: **Marks: 15**

General Measure Theory: Measure space, completion of a measure space, signed measure, Hahn and Jordan decomposition theorem for finite signed measure, extension of a premeasure to a measure, Carathéodory-Hahn theorem, Randon-Nikodym theorem for finite and σ -finite signed measures and its consequences, computation of the Randon-Nikodym derivative, Nikodym metric space, Vitali-Hahn-Saks Theorem.

Unit-II: **Marks: 15**

General L^P Space: Completeness, duality and weak convergence.

Unit-III: **Marks: 20**

Product measures: Fubini and Tonelli theorem, Lebesgue measure on Euclidean space $\mathbb{R}^n$ Lebesgue-Stieltjes measures and Lebesgue-Stieltjes integral, Hausdorff measure and its dimension.

Unit-IV: **Marks: 20**

Measure and Topology: Construction of Radon measure, Riesz-Markov theorem, Riesz representation theorem for the dual of C(X), regularity of Baire measures, Borel measure on compact groups, Von Neumann’s theorem, Bogoliubov-Krilov theorem.

Text Books:

1. Royden,H. L., Fitzpatrick, P. M., Real Analysis, 4thEdition, PHI Learning Private Limited, New Delhi, 2011

Reference Books:

1. Munroe, M. E.: Introductions to measure and integration, Addison Wesley, 1953.
2. Berberian, S. K.: Measure and integration, Chelsa Pub. Co. N.Y. 1965.
3. Ruddin, W.: Real and Complex Analysis, Tata McGraw Hill, New Delhi, 1974

SEMESTER - II

Paper Code: MATSPL 25094 (B)
Paper Title: Biomathematics
Theory Marks: 70
Internal Marks: 30

Course Learning Objectives:

1. Comprehend continuous population models for single species, including exponential and logistic growth models, and their qualitative analysis.
2. Learn to perform stability and equilibrium analysis of various population growth models, including those with delays.
3. Understand and model interactions between species using continuous models, focusing on competition, mutualism, and predator-prey dynamics.
4. Gain insights into the impact of harvesting on natural populations, including models with delayed recruitment and age distribution.
5. Utilize mathematical techniques such as differential equations and qualitative analysis to study and solve population dynamics problems.

Course Learning Outcomes:

1. Demonstrate the ability to model single-species population growth using exponential, Malthusian, and logistic growth models, and perform qualitative analysis of these models.
2. Analyze continuous single-species population models with delays, understand the general delay model, and perform stability and equilibrium analysis.
3. Apply continuous models to study interactions between species, including competition, mutualism, and predator-prey dynamics using Lotka-Volterra models.
4. Evaluate the effects of different harvesting strategies on population dynamics, including constant effort and constant yield harvesting, and analyze models with age distribution.
5. Analyze predator-prey interactions using Lotka-Volterra models, understand the role of density dependence, and study classic laboratory experiments and natural system predation.

Unit-I: **Marks: 20**

Continuous population Model for single species, Exponential population growth model, continuous population growth model: Malthus model for population growth, General population growth model, Qualitative Analysis: Equilibrium points, Stability Analysis, Logistic population growth model and their qualitative analysis, logistic growth model for non-isolated population.

Unit-II: **Marks: 20**

Insect Out-break model: Spruce Budworm, Continuous Single species population model with Delays: Introduction, General Delay model and Qualitative Analysis, Logistic model with time delay effects, Definition of stability, equilibrium points & stability Analysis.

Unit-III: **Marks: 10**

Continuous Models for Interacting Population: Interaction between species: two species models, community matrix approach, Qualitative behaviour of community matrix, Competition: Lotka-Volterra models, Competition Models, Principle of competitive exclusion, Models for Mutualism.

Unit-IV: **Marks: 10**

Harvesting a single Natural population: Harvesting in Delayed recruitment models: Constant effort Harvesting, constant yield harvesting, population model with Age Distribution, Simple Discrete population model.

Unit-V: **Marks: 10**

Predator: Prey interaction: Lotka-Volterra models, dynamic of simple Lotka-Volterra models, Role of density dependent in the Prey, Classic laboratory experiment on predator, predation in natural system. Some predator- prey models.

Reference Books:

[1] J. N. Kapur, Mathematical Modelling, New Age International Publishers.
[2] J.D. Murray Mathematical Biology (An Introduction, Vol. I & II), Springer- Verlag.
[3] J.N. Kapur, Mathematical Model in Biology and Medicines.
[4] S. I. Rubinow, Introduction to Mathematical Biology, John Wiley andSons, 1975.
[5] MA Khanday, Introduction to Modeling and Biomathematics, Dilpreet Publishers New Delhi, 2016.
[6] Jaffrey R. Chasnov, Mathematical Biology, Hong Kong Press.

SEMESTER - II

Paper Code: MATSPL 25104 (A)
Paper Title: Network Theory
Theory Marks: 70
Internal Marks: 30

Course Outcome: This course is about the networks involving graphs. The course deal with various types of complex and random networks. This course also included the basic of Graph neural networks.

Unit-I: **Marks: 15**

Overview of graph network, Network Flow: Max-flow Min-cut and the Ford-Fulkerson algorithm.

Unit-II: **Marks: 20**

Complex Networks: Introduction to complex networks, Network traversal, Construction of Euler tour, Finding a Hamilton cycle, Trees in transportation networks, Routing in communication networks.

Unit-III: **Marks: 10**

Random networks, Computer networks; Social networks analysis; Structural balance; Affiliation networks; Equivalence; Structural equivalence.

Unit-IV: **Marks: 25**

Representation Learning, Graph representation learning, Introduction to Graph Neural network (GNN), Convolutional neural network (CNN), General Framework of GNN and CNN, Types of GNN: Recurrent Graph Neural Network (RECGNN or RGNN), Graph Attention Network (GAN), Applications of GNN.

Books:

[1] Introduction to Graph Theory, D. B. West, Prentice-Hall, (2001).
[2] Graph Theory and Complex Networks: An Introduction, Maarten van Steen, (2010).
[3] Networks, Crowds, and Markets: Reasoning about a Highly Connected World, David Easley and Jon Kleinberg. Cambridge University Press, (2010).
[4] Introduction to Graph Neural Networks, Zhiyuan Liu and Zhou Jie, Springer Nature, (2022).
[5] Graph Neural Networks: Foundations, Frontiers, and Applications, Jian Pei (Editor), Liang Zhao (Editor), Lingfei Wu (Editor), Peng Cui (Editor), Springer, (2022).
[6] Graph Representation Learning, William L. Hamilton, McGill University, (2020).

SEMESTER - II

Paper Code: MATSPL 25104 (B)
Paper Title: Fluid Dynamics II
Theory Marks: 70
Internal Marks: 30

Course Learning Objectives:

1. Gain a comprehensive understanding of wave dynamics, including the behavior of waves at the interface of two liquids, energy transmission in waves, and the significance of group velocity.
2. Develop a thorough understanding of the Navier-Stokes equations for viscous fluid motion, and study various types of steady laminar flows, including flows between parallel plates and through tubes.
3. Learn the principles of dynamical similarity and dimensional analysis, including the Buckingham Pi-theorem, and understand the significance of Reynolds number and magnetic Reynolds number.
4. Explore the theory of very slow motions, including Stokes and Oseen's equations, and apply these to analyze flow past a sphere and lubrication theory.
5. Gain insights into the theory of laminar boundary layers, including the two-dimensional boundary layer equations for flow over a plane wall, and apply Blasius-Topfer solutions to solve boundary layer problems.

Course Learning Outcomes:

1. Demonstrate the ability to analyze energy in stationary and progressive waves, calculate group velocity, and understand its dynamical significance in simple harmonic surface waves.
2. Apply the Navier-Stokes equations to solve problems involving steady laminar flow between parallel plates, through circular and non-circular tubes, and understand various flow scenarios such as Plane Couette and Poiseuille flow.
3. Utilize principles of dimensional analysis and the Buckingham Pi-theorem to solve fluid dynamics problems, and explain the significance of Reynolds number and its applications in different fluid flow conditions.
4. Solve problems involving very slow motions using Stokes and Oseen's equations, analyze flow past a sphere, and apply lubrication theory to practical fluid dynamics problems.
5. Apply the theory of laminar boundary layers to two-dimensional flow over a plane wall, use Blasius-Topfer solutions to find solutions to boundary layer equations, and understand the implications for practical fluid dynamics scenarios.

Unit-I: **Marks: 20**

Waves, Waves at the interface of two liquids, and with upper surface free, Energy of Stationary Waves and Progressive Waves, Group Velocity, Rate of Transmission of energy in simple harmonic surface waves, Dynamical significance of group velocity.

Unit-II: **Marks: 20**

Navier-Stoke's equations of motion of a viscous fluid, Equation of state for perfect fluid, Steady laminar flow between two parallel plates, Plane Coutte flow, Plane Poiseuille flow, The Hagen-Poiseuille flow, Laminar steady flow between two coaxial circular cylinders. Laminar steady flow of incompressible viscous fluid in tubes of cross-section other than circular.

Unit-III: **Marks: 15**

Dynamical similarity; Dimensional Analysis, Buckingham Pi-theorem; Reynold's number; Significance of Reynold's number, Maxwell's electromagnetic field equation; Magnetic Reynold's number.

Unit-IV: **Marks: 15**

Theory of very slow motions, Stokes equation, Oseen's equations, flow past a sphere, Lubrication theory. Theory of laminar boundary layers, Two-dimensional boundary layer equations for flow over a plane wall, Blasious-Topfer solutions.

Reference Books:

[1] Chorlton, Text Book of Fluid Dynamics, CBS Publishers, Delhi,1985.
[2] G. K. Batchelor, An Introduction to Fluid Mechanics, Foundation Books, New Delhi, 1994.
[3] H. Schlichting, Boundary Layer Theory, McGraw Hill Book Company, New York, 1971.
[4] M. D. Raisinghania, Fluid Mechanics (With Hydrodynamics) S. Chand and Company Ltd., New Delhi.
[5] R. K. Rathy, An Introduction to Fluid Dynamics, Oxford and IBH Publishing Company, New Delhi, 1976.